Package ‘multvardiv’

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Type Package

Title Multivariate Probability Distributions, Statistical Divergence

Version 1.0.10

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Description Multivariate generalized Gaussian distribution,
Multivariate Cauchy distribution,
Multivariate t distribution.
Distance between two distributions (see N. Bouh-
lel and A. Dziri (2019): <doi:10.1109/LSP.2019.2915000>,
N. Bouhlel and D. Rousseau (2022): <doi:10.3390/e24060838>,
N. Bouhlel and D. Rousseau (2023): <doi:10.1109/LSP.2023.3324594>).
Manipulation of these multivariate probability distributions.

Depends R (>= 4.4.0)

Imports rgl, MASS, data.table

License GPL (>= 3)

URL https://forgemia.inra.fr/imhorphen/multvardiv

BugReports https://forgemia.inra.fr/imhorphen/multvardiv/-/issues

Encoding UTF-8

RoxygenNote 7.3.2

Suggests testthat (>= 3.2.1)

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NeedsCompilation no

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Repository CRAN

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Contents

`contourmvd` *Contour Plot of a Bivariate Density*

Description

Contour plot of the probability density of a multivariate distribution with 2 variables:

- generalized Gaussian distribution (MGGD) with mean vector `mu`, dispersion matrix `Sigma` and shape parameter `beta`
- Cauchy distribution (MCD) with location parameter `mu` and scatter matrix `Sigma`
- t distribution (MTD) with location parameter `mu`, scatter matrix `Sigma` and degrees of freedom `nu`

This function uses the `contour` function.

Usage

```
contourmvd(mu, Sigma, beta = NULL, nu = NULL,
                  distribution = c("mggd", "mcd", "mtd"),
                  xlim = c(mu[1] + c(-10, 10)*Sigma[1, 1]),
                  ylim = c(mu[2] + c(-10, 10)*Sigma[2, 2]),
                  zlim = NULL, npt = 30, nx = npt, ny = npt,
                  main = NULL, sub = NULL, nlevels = 10,
                  levels = pretty(zlim, nlevels), tol = 1e-6, ...)
```

Arguments

mu	length 2 numeric vector.
Sigma	symmetric, positive-definite square matrix of order 2. The dispersion matrix.
beta	numeric. If distribution = "mggd", the shape parameter of the MGGD. NULL if dist is "mcd" or "mtd".
nu	numeric. If distribution = "mtd", the degrees of freedom of the MTD. NULL if distribution is "mggd" or "mcd".
distribution	character string. The probability distribution. It can be "mggd" (multivariate generalized Gaussian distribution) "mcd" (multivariate Cauchy) or "mtd" (multivariate t).
xlim, ylim	x-and y- limits.
zlim	z- limits. If NULL, it is the range of the values of the density on the x and y values within xlim and ylim.
npt	number of points for the discretisation.
nx, ny	number of points for the discretisation among the x- and y- axes.
main, sub	main and sub title, as for title. If omitted, the main title is set to "Multivariate generalised Gaussian density", "Multivariate Cauchy density" or "Multivariate t density".
nlevels, levels	arguments to be passed to the contour function.
tol	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma, for the estimation of the density. See dmggd, dmcd or dmtd.
...	additional arguments to plot.window, title, Axis and box, typically graphical parameters such as cex.axis.

Value

Returns invisibly the probability density function.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

S. Kotz and Saralees Nadarajah (2004), Multivariate t Distributions and Their Applications, Cambridge University Press.

See Also

plotmvd: plot of a bivariate generalised Gaussian, Cauchy or t density.

dmggd: probability density of a multivariate generalised Gaussian distribution.

dmcd: probability density of a multivariate Cauchy distribution.

dmtd: probability density of a multivariate t distribution.

Examples

```
mu <- c(1, 4)
Sigma <- matrix(c(0.8, 0.2, 0.2, 0.2), nrow = 2)

# Bivariate generalized Gaussian distribution
beta <- 0.74
contourmvd(mu, Sigma, beta = beta, distribution = "mggd")

# Bivariate Cauchy distribution
contourmvd(mu, Sigma, distribution = "mcd")

# Bivariate t distribution
nu <- 1
contourmvd(mu, Sigma, nu = nu, distribution = "mtd")
```

`diststudent` *Distance/Divergence between Centered Multivariate t Distributions*

Description

Computes the distance or divergence (Renyi divergence, Bhattacharyya distance or Hellinger distance) between two random vectors distributed according to multivariate t distributions (MTD) with zero mean vector.

Usage

```
diststudent(nu1, Sigma1, nu2, Sigma2,
                   dist = c("renyi", "bhattacharyya", "hellinger"),
                   bet = NULL, eps = 1e-06)
```

Arguments

`nu1`	numeric. The degrees of freedom of the first distribution.
`Sigma1`	symmetric, positive-definite matrix. The correlation matrix of the first distribution.
`nu2`	numeric. The degrees of freedom of the second distribution.
`Sigma2`	symmetric, positive-definite matrix. The correlation matrix of the second distribution.
`dist`	character. The distance or divergence used. One of `"renyi"` (default), `"battacharyya"` or `"hellinger"`.
`bet`	numeric, positive and not equal to 1. Order of the Renyi divergence. Ignored if `distance="bhattacharyya"` or `distance="hellinger"`.
`eps`	numeric. Precision for the computation of the partial derivative of the Lauricella D-hypergeometric function (see Details). Default: 1e-06.

Details

Given X_1, a random vector of $\mathbb{R}^p$ distributed according to the MTD with parameters $(\nu_1, \mathbf{0}, \Sigma_1)$ and X_2, a random vector of $\mathbb{R}^p$ distributed according to the MTD with parameters $(\nu_2, \mathbf{0}, \Sigma_2)$.

Let $\delta_1 = \frac{\nu_1+p}{2}\beta$, $\delta_2 = \frac{\nu_2+p}{2}(1-\beta)$ and $\lambda_1, \dots, \lambda_p$ the eigenvalues of the square matrix $\Sigma_1\Sigma_2^{-1}$ sorted in increasing order:

$$\lambda_1 < \dots < \lambda_{p-1} < \lambda_p$$

The Renyi divergence between X_1 and X_2 is:

$$D_R^\beta(\mathbf{X}_1||\mathbf{X}_1) = \frac{1}{\beta-1}\Bigg[\beta \ln\left(\frac{\Gamma\left(\frac{\nu_1+p}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)\nu_2^{\frac{p}{2}}}{\Gamma\left(\frac{\nu_2+p}{2}\right)\Gamma\left(\frac{\nu_1}{2}\right)\nu_1^{\frac{p}{2}}}\right) + \ln\left(\frac{\Gamma\left(\frac{\nu_2+p}{2}\right)}{\Gamma\left(\frac{\nu_2}{2}\right)}\right) + \ln\left(\frac{\Gamma\left(\delta_1+\delta_2-\frac{p}{2}\right)}{\Gamma(\delta_1+\delta_2)}\right) - \frac{\beta}{2}\sum_{i=1}^{p}\ln\lambda_i + \ln F_D\Bigg]$$

with F_D given by:

- If $\dfrac{\nu_1}{\nu_2}\lambda_1 > 1$:

$$F_D = F_D^{(p)}\Bigg(\delta_1, \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_{p}; \delta_1+\delta_2; 1-\frac{\nu_2}{\nu_1\lambda_1}, \dots, 1-\frac{\nu_2}{\nu_1\lambda_p}\Bigg)$$

- If $\dfrac{\nu_1}{\nu_2}\lambda_p < 1$:

$$F_D = \prod_{i=1}^{p}\left(\frac{\nu_1}{\nu_2}\lambda_i\right)^{\frac{1}{2}} F_D^{(p)}\Bigg(\delta_2, \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_{p}; \delta_1+\delta_2; 1-\frac{\nu_1}{\nu_2}\lambda_1, \dots, 1-\frac{\nu_1}{\nu_2}\lambda_p\Bigg)$$

- If $\dfrac{\nu_1}{\nu_2}\lambda_1 < 1$ and $\dfrac{\nu_1}{\nu_2}\lambda_p > 1$:

$$F_D = \left(\frac{\nu_2}{\nu_1}\frac{1}{\lambda_p}\right)^{\delta_2}\prod_{i=1}^{p}\left(\frac{\nu_1}{\nu_2}\lambda_i\right)^{\frac{1}{2}} F_D^{(p)}\Bigg(\delta_2, \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_{p}, \delta_1+\delta_2-\frac{p}{2}; \delta_1+\delta 2; 1-\frac{\lambda_1}{\lambda_p}, \dots, 1-\frac{\lambda_{p-1}}{\lambda_p}, 1-\frac{\nu_2}{\nu_1}\frac{1}{\lambda_p}\Bigg)$$

where $F_D^{(p)}$ is the Lauricella D-hypergeometric function defined for p variables:

$$F_D^{(p)}\left(a; b_1, ..., b_p; g; x_1, ..., x_p\right) = \sum_{m_1 \geq 0} ... \sum_{m_p \geq 0} \frac{(a)_{m_1+...+m_p}(b_1)_{m_1} ... (b_p)_{m_p}}{(g)_{m_1+...+m_p}} \frac{x_1^{m_1}}{m_1!} ... \frac{x_p^{m_p}}{m_p!}$$

Its computation uses the `lauricella` function.

The Bhattacharyya distance is given by:

$$D_B(\mathbf{X}_1||\mathbf{X}_2) = \frac{1}{2}D_R^{1/2}(\mathbf{X}_1||\mathbf{X}_2)$$

And the Hellinger distance is given by:

$$D_H(\mathbf{X}_1||\mathbf{X}_2) = 1 - \exp\left(-\frac{1}{2}D_R^{1/2}(\mathbf{X}_1||\mathbf{X}_2)\right)$$

Value

A numeric value: the divergence between the two distributions, with two attributes `attr(, "epsilon")` (precision of the result of the Lauricella D-hypergeometric function,see Details) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

N. Bouhlel and D. Rousseau (2023), Exact Rényi and Kullback-Leibler Divergences Between Multivariate t-Distributions, IEEE Signal Processing Letters. doi:10.1109/LSP.2023.3324594

Examples

```
nu1 <- 2
Sigma1 <- matrix(c(2, 1.2, 0.4, 1.2, 2, 0.6, 0.4, 0.6, 2), nrow = 3)
nu2 <- 4
Sigma2 <- matrix(c(1, 0.3, 0.1, 0.3, 1, 0.4, 0.1, 0.4, 1), nrow = 3)

# Renyi divergence
diststudent(nu1, Sigma1, nu2, Sigma2, bet = 0.25)
diststudent(nu2, Sigma2, nu1, Sigma1, bet = 0.25)

# Bhattacharyya distance
diststudent(nu1, Sigma1, nu2, Sigma2, dist = "bhattacharyya")
diststudent(nu2, Sigma2, nu1, Sigma1, dist = "bhattacharyya")

# Hellinger distance
diststudent(nu1, Sigma1, nu2, Sigma2, dist = "hellinger")
diststudent(nu2, Sigma2, nu1, Sigma1, dist = "hellinger")
```

---

dmcd *Density of a Multivariate Cauchy Distribution*

Description

Density of the multivariate (p variables) Cauchy distribution (MCD) with location parameter `mu` and scatter matrix `Sigma`.

Usage

```
dmcd(x, mu, Sigma, tol = 1e-6)
```

Arguments

`x`	length p numeric vector.
`mu`	length p numeric vector. The location parameter.
`Sigma`	symmetric, positive-definite square matrix of order p. The scatter matrix.
`tol`	tolerance (relative to largest eigenvalue) for numerical lack of positive-definiteness in Sigma.

Details

The density function of a multivariate Cauchy distribution is given by:

$$f(\mathbf{x}|\boldsymbol{\mu}, \Sigma) = \frac{\Gamma\left(\frac{1+p}{2}\right)}{\pi^{p/2}\Gamma\left(\frac{1}{2}\right)|\Sigma|^{\frac{1}{2}}\left[1 + (\mathbf{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right]^{\frac{1+p}{2}}}$$

Value

The value of the density.

Author(s)

Pierre Santagostini, Nizar Bouhlel

See Also

`rmcd`: random generation from a MCD.

`estparmcd`: estimation of the parameters of a MCD.

`plotmvd`, `contourmvd`: plot of the probability density of a bivariate distribution.

Examples

```
mu <- c(0, 1, 4)
sigma <- matrix(c(1, 0.6, 0.2, 0.6, 1, 0.3, 0.2, 0.3, 1), nrow = 3)
dmcd(c(0, 1, 4), mu, sigma)
dmcd(c(1, 2, 3), mu, sigma)
```

---

dmggd — *Density of a Multivariate Generalized Gaussian Distribution*

Description

Density of the multivariate (p variables) generalized Gaussian distribution (MGGD) with mean vector `mu`, dispersion matrix `Sigma` and shape parameter `beta`.

Usage

```
dmggd(x, mu, Sigma, beta, tol = 1e-6)
```

Arguments

x	length p numeric vector.
mu	length p numeric vector. The mean vector.
Sigma	symmetric, positive-definite square matrix of order p. The dispersion matrix.
beta	positive real number. The shape of the distribution.
tol	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma.

Details

The density function of a multivariate generalized Gaussian distribution is given by:

$$f(\mathbf{x}|\boldsymbol{\mu}, \Sigma, \beta) = \frac{\Gamma\left(\frac{p}{2}\right)}{\pi^{\frac{p}{2}}\Gamma\left(\frac{p}{2\beta}\right)2^{\frac{p}{2\beta}}}\frac{\beta}{|\Sigma|^{\frac{1}{2}}}e^{-\frac{1}{2}\left((\mathbf{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)^\beta}$$

When $p = 1$ (univariate case) it becomes:

$$f(x|\mu, \sigma, \beta) = \frac{\beta}{\Gamma\left(\frac{1}{2\beta}\right)2^{\frac{1}{2\beta}}\sqrt{\sigma}}\, e^{-\frac{1}{2}\left(\frac{(x-\mu)^2}{\sigma}\right)^\beta}$$

Value

The value of the density.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

See Also

rmggd: random generation from a MGGD.

estparmggd: estimation of the parameters of a MGGD.

plotmvd, contourmvd: plot of the probability density of a bivariate distribution.

Examples

```
mu <- c(0, 1, 4)
Sigma <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
beta <- 0.74
dmggd(c(0, 1, 4), mu, Sigma, beta)
dmggd(c(1, 2, 3), mu, Sigma, beta)
```

dmtd *Density of a Multivariate t Distribution*

Description

Density of the multivariate (p variables) t distribution (MTD) with degrees of freedom `nu`, mean vector `mu` and correlation matrix `Sigma`.

Usage

```
dmtd(x, nu, mu, Sigma, tol = 1e-6)
```

Arguments

`x`	length p numeric vector.
`nu`	numeric. The degrees of freedom.
`mu`	length p numeric vector. The mean vector.
`Sigma`	symmetric, positive-definite square matrix of order p. The correlation matrix.
`tol`	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma.

Details

The density function of a multivariate t distribution with p variables is given by:

$$f(\mathbf{x}|\nu, \boldsymbol{\mu}, \Sigma) = \frac{\Gamma\left(\frac{\nu+p}{2}\right)|\Sigma|^{-1/2}}{\Gamma\left(\frac{\nu}{2}\right)(\nu\pi)^{p/2}}\left(1 + \frac{1}{\nu}(\mathbf{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)^{-\frac{\nu+p}{2}}$$

When $p = 1$ (univariate case) it is the location-scale t distribution, with density function:

$$f(x|\nu, \mu, \sigma^2) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi\sigma^2}}\left(1 + \frac{(x-\mu)^2}{\nu\sigma^2}\right)^{-\frac{\nu+1}{2}}$$

Value

The value of the density.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

S. Kotz and Saralees Nadarajah (2004), Multivariate t Distributions and Their Applications, Cambridge University Press.

See Also

rmtd: random generation from a MTD.

estparmtd: estimation of the parameters of a MTD.

plotmvd, contourmvd: plot of the probability density of a bivariate distribution.

Examples

```
nu <- 1
mu <- c(0, 1, 4)
Sigma <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
dmtd(c(0, 1, 4), nu, mu, Sigma)
dmtd(c(1, 2, 3), nu, mu, Sigma)

# Univariate
dmtd(1, 3, 0, 1)
dt(1, 3)
```

---

estparmcd *Estimation of the Parameters of a Multivariate Cauchy Distribution*

Description

Estimation of the mean vector and correlation matrix of a multivariate Cauchy distribution (MCD).

Usage

```
estparmcd(x, eps = 1e-6)
```

Arguments

x	numeric matrix or data frame.
eps	numeric. Precision for the estimation of the parameters.

Details

The EM method is used to estimate the parameters.

Value

A list of 2 elements:

- mu the mean vector.
- Sigma: symmetric positive-definite matrix. The correlation matrix.

with two attributes attr(, "epsilon") (precision of the result) and attr(, "k") (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

Doğru, F., Bulut, Y. M. and Arslan, O. (2018). Doubly reweighted estimators for the parameters of the multivariate t-distribution. Communications in Statistics - Theory and Methods. 47. doi:10.1080/03610926.2018.1445861.

See Also

dmcd: probability density of a MTD

rmcd: random generation from a MTD.

Examples

```
mu <- c(0, 1, 4)
Sigma <- matrix(c(1, 0.6, 0.2, 0.6, 1, 0.3, 0.2, 0.3, 1), nrow = 3)
x <- rmcd(100, mu, Sigma)

# Estimation of the parameters
estparmcd(x)
```

`estparmggd` *Estimation of the Parameters of a Multivariate Generalized Gaussian Distribution*

Description

Estimation of the mean vector, dispersion matrix and shape parameter of a multivariate generalized Gaussian distribution (MGGD).

Usage

```
estparmggd(x, eps = 1e-6, display = FALSE, plot = display)
```

Arguments

`x`	numeric matrix or data frame.
`eps`	numeric. Precision for the estimation of the beta parameter.
`display`	logical. When `TRUE` the value of the `beta` parameter at each iteration is printed.
`plot`	logical. When `TRUE` the successive values of the `beta` parameter are plotted, allowing to visualise its convergence.

Details

The μ parameter is the mean vector of x.

The dispersion matrix Σ and shape parameter β are computed using the method presented in Pascal et al., using an iterative algorithm.

The precision for the estimation of beta is given by the eps parameter.

Value

A list of 3 elements:

- mu the mean vector.
- Sigma: symmetric positive-definite matrix. The dispersion matrix.
- beta non-negative numeric value. The shape parameter.

with two attributes attr(, "epsilon") (precision of the result) and attr(, "k") (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

F. Pascal, L. Bombrun, J.Y. Tourneret, Y. Berthoumieu. Parameter Estimation For Multivariate Generalized Gaussian Distribution. IEEE Trans. Signal Processing, vol. 61 no. 23, p. 5960-5971, Dec. 2013. doi:10.1109/TSP.2013.2282909

See Also

dmggd: probability density of a MGGD.

rmggd: random generation from a MGGD.

Examples

```
mu <- c(0, 1, 4)
Sigma <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
beta <- 0.74
x <- rmggd(100, mu, Sigma, beta)

# Estimation of the parameters
estparmggd(x)
```

estparmtd — *Estimation of the Parameters of a Multivariate t Distribution*

Description

Estimation of the degrees of freedom, mean vector and correlation matrix of a multivariate t distribution (MTD).

Usage

```
estparmtd(x, eps = 1e-6, display = FALSE, plot = display)
```

Arguments

`x`	numeric matrix or data frame.
`eps`	numeric. Precision for the estimation of the parameters.
`display`	logical. When `TRUE` the value of the `nu` parameter at each iteration is printed.
`plot`	logical. When `TRUE` the successive values of the `nu` parameter are plotted, allowing to visualise its convergence.

Details

The EM method is used to estimate the parameters.

Value

A list of 3 elements:

- `nu` non-negative numeric value. The degrees of freedom.
- `mu` the mean vector.
- `Sigma`: symmetric positive-definite matrix. The correlation matrix.

with two attributes `attr(, "epsilon")` (precision of the result) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

Doğru, F., Bulut, Y. M. and Arslan, O. (2018). Doubly reweighted estimators for the parameters of the multivariate t-distribution. Communications in Statistics - Theory and Methods. 47. doi:10.1080/03610926.2018.1445861.

See Also

dmtd: probability density of a MTD

rmtd: random generation from a MTD.

Examples

```
nu <- 3
mu <- c(0, 1, 4)
Sigma <- matrix(c(1, 0.6, 0.2, 0.6, 1, 0.3, 0.2, 0.3, 1), nrow = 3)
x <- rmtd(100, nu, mu, Sigma)

# Estimation of the parameters
estparmggd(x)
```

---

kld — *Kullback-Leibler Divergence between Centered Multivariate Distributions*

Description

Computes the Kullback-Leibler divergence between two random vectors distributed according to centered multivariate distributions:

- multivariate generalized Gaussian distribution (MGGD) with zero mean vector, using the kldggd function
- multivariate Cauchy distribution (MCD) with zero location vector, using the kldcauchy function
- multivariate t distribution (MTD) with zero mean vector, using the kldstudent function

One can also use one of the kldggd, kldcauchy or kldstudent functions, depending on the probability distribution.

Usage

```
kld(Sigma1, Sigma2, distribution = c("mggd", "mcd", "mtd"),
            beta1 = NULL, beta2 = NULL, nu1 = NULL, nu2 = NULL, eps = 1e-06)
```

Arguments

Sigma1	symmetric, positive-definite matrix. The scatter matrix of the first distribution.
Sigma2	symmetric, positive-definite matrix. The scatter matrix of the second distribution.
distribution	the probability distribution. It can be "mggd" (multivariate generalized Gaussian distribution) "mcd" (multivariate Cauchy) or "mtd" (multivariate t).
beta1, beta2	numeric. If distribution = "mggd", the shape parameters of the first and second distributions. NULL if distribution is "mcd" or "mtd".

nu1, nu2	numeric. If distribution = "mtd", the degrees of freedom of the first and second distributions. NULL if distribution is "mggd" or "mcd".
eps	numeric. Precision for the computation of the Lauricella D-hypergeometric function if distribution is "mggd" (see kldggd) or of its partial derivative if distribution = "mcd" or distribution = "mtd" (see kldcauchy or kldstudent). Default: 1e-06.

Value

A numeric value: the Kullback-Leibler divergence between the two distributions, with two attributes attr(, "epsilon") (precision of the Lauricella D-hypergeometric function or of its partial derivative) and attr(, "k") (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

N. Bouhlel, A. Dziri, Kullback-Leibler Divergence Between Multivariate Generalized Gaussian Distributions. IEEE Signal Processing Letters, vol. 26 no. 7, July 2019. doi:10.1109/LSP.2019.2915000

N. Bouhlel, D. Rousseau, A Generic Formula and Some Special Cases for the Kullback–Leibler Divergence between Central Multivariate Cauchy Distributions. Entropy, 24, 838, July 2022. doi:10.3390/e24060838

N. Bouhlel and D. Rousseau (2023), Exact Rényi and Kullback-Leibler Divergences Between Multivariate t-Distributions, IEEE Signal Processing Letters. doi:10.1109/LSP.2023.3324594

Examples

```
# Generalized Gaussian distributions
beta1 <- 0.74
beta2 <- 0.55
Sigma1 <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
Sigma2 <- matrix(c(1, 0.3, 0.2, 0.3, 0.5, 0.1, 0.2, 0.1, 0.7), nrow = 3)
# Kullback-Leibler divergence
kl12 <- kld(Sigma1, Sigma2, "mggd", beta1 = beta1, beta2 = beta2)
kl21 <- kld(Sigma2, Sigma1, "mggd", beta1 = beta2, beta2 = beta1)
print(kl12)
print(kl21)
# Distance (symmetrized Kullback-Leibler divergence)
kldist <- as.numeric(kl12) + as.numeric(kl21)
print(kldist)

# Cauchy distributions
Sigma1 <- matrix(c(1, 0.6, 0.2, 0.6, 1, 0.3, 0.2, 0.3, 1), nrow = 3)
Sigma2 <- matrix(c(1, 0.3, 0.1, 0.3, 1, 0.4, 0.1, 0.4, 1), nrow = 3)
kld(Sigma1, Sigma2, "mcd")
kld(Sigma2, Sigma1, "mcd")

Sigma1 <- matrix(c(0.5, 0, 0, 0, 0.4, 0, 0, 0, 0.3), nrow = 3)
```

```
Sigma2 <- diag(1, 3)
# Case when all eigenvalues of Sigma1 %*% solve(Sigma2) are < 1
kld(Sigma1, Sigma2, "mcd")
# Case when all eigenvalues of Sigma1 %*% solve(Sigma2) are > 1
kld(Sigma2, Sigma1, "mcd")

# Student distributions
nu1 <- 2
Sigma1 <- matrix(c(2, 1.2, 0.4, 1.2, 2, 0.6, 0.4, 0.6, 2), nrow = 3)
nu2 <- 4
Sigma2 <- matrix(c(1, 0.3, 0.1, 0.3, 1, 0.4, 0.1, 0.4, 1), nrow = 3)
# Kullback-Leibler divergence
kld(Sigma1, Sigma2, "mtd", nu1 = nu1, nu2 = nu2)
kld(Sigma2, Sigma1, "mtd", nu1 = nu2, nu2 = nu1)
```

---

kldcauchy — *Kullback-Leibler Divergence between Centered Multivariate Cauchy Distributions*

Description

Computes the Kullback-Leibler divergence between two random vectors distributed according to multivariate Cauchy distributions (MCD) with zero location vector.

Usage

```
kldcauchy(Sigma1, Sigma2, eps = 1e-06)
```

Arguments

Sigma1	symmetric, positive-definite matrix. The scatter matrix of the first distribution.
Sigma2	symmetric, positive-definite matrix. The scatter matrix of the second distribution.
eps	numeric. Precision for the computation of the partial derivative of the Lauricella D-hypergeometric function (see Details). Default: 1e-06.

Details

Given X_1, a random vector of $\mathbb{R}^p$ distributed according to the MCD with parameters $(0, \Sigma_1)$ and X_2, a random vector of $\mathbb{R}^p$ distributed according to the MCD with parameters $(0, \Sigma_2)$.

Let $\lambda_1, \dots, \lambda_p$ the eigenvalues of the square matrix $\Sigma_1 \Sigma_2^{-1}$ sorted in increasing order:

$$\lambda_1 < \dots < \lambda_{p-1} < \lambda_p$$

Depending on the values of these eigenvalues, the computation of the Kullback-Leibler divergence of X_1 from X_2 is given by:

$$KL(X_1||X_2) = -\frac{1}{2}\ln\prod_{i=1}^{p}\lambda_i + \frac{1+p}{2}D$$

where D is given by:

- if $\lambda_1 < 1$ and $\lambda_p > 1$:

$$D = \ln\lambda_p - \frac{\partial}{\partial a}\left\{F_D^{(p)}\left(a, \underbrace{\frac{1}{2},\dots,\frac{1}{2}, a+\frac{1}{2}}_{p}; a+\frac{1+p}{2}; 1-\frac{\lambda_1}{\lambda_p},\dots,1-\frac{\lambda_{p-1}}{\lambda_p}, 1-\frac{1}{\lambda_p}\right)\right\}\Big|_{a=0}$$

- if $\lambda_p < 1$:

$$D = \frac{\partial}{\partial a}\left\{F_D^{(p)}\left(a, \underbrace{\frac{1}{2},\dots,\frac{1}{2}}_{p}; a+\frac{1+p}{2}; 1-\lambda_1,\dots,1-\lambda_p\right)\right\}\Big|_{a=0}$$

- if $\lambda_1 > 1$:

$$D = \prod_{i=1}^{p}\frac{1}{\sqrt{\lambda_i}} \times \frac{\partial}{\partial a}\left\{F_D^{(p)}\left(\frac{1+p}{2}, \underbrace{\frac{1}{2},\dots,\frac{1}{2}}_{p}; a+\frac{1+p}{2}; 1-\frac{1}{\lambda_1},\dots,1-\frac{1}{\lambda_p}\right)\right\}\Big|_{a=0}$$

$F_D^{(p)}$ is the Lauricella D-hypergeometric function defined for p variables:

$$F_D^{(p)}(a; b_1, ..., b_p; g; x_1, ..., x_p) = \sum_{m_1 \geq 0} ... \sum_{m_p \geq 0} \frac{(a)_{m_1+...+m_p}(b_1)_{m_1}...(b_p)_{m_p}}{(g)_{m_1+...+m_p}} \frac{x_1^{m_1}}{m_1!} ... \frac{x_p^{m_p}}{m_p!}$$

Value

A numeric value: the Kullback-Leibler divergence between the two distributions, with two attributes `attr(, "epsilon")` (precision of the partial derivative of the Lauricella D-hypergeometric function,see Details) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

N. Bouhlel, D. Rousseau, A Generic Formula and Some Special Cases for the Kullback–Leibler Divergence between Central Multivariate Cauchy Distributions. Entropy, 24, 838, July 2022. doi:10.3390/e24060838

Examples

```
Sigma1 <- matrix(c(1, 0.6, 0.2, 0.6, 1, 0.3, 0.2, 0.3, 1), nrow = 3)
Sigma2 <- matrix(c(1, 0.3, 0.1, 0.3, 1, 0.4, 0.1, 0.4, 1), nrow = 3)
kldcauchy(Sigma1, Sigma2)
kldcauchy(Sigma2, Sigma1)

Sigma1 <- matrix(c(0.5, 0, 0, 0, 0.4, 0, 0, 0, 0.3), nrow = 3)
Sigma2 <- diag(1, 3)
# Case when all eigenvalues of Sigma1 %*% solve(Sigma2) are < 1
kldcauchy(Sigma1, Sigma2)
# Case when all eigenvalues of Sigma1 %*% solve(Sigma2) are > 1
kldcauchy(Sigma2, Sigma1)
```

`kldggd` *Kullback-Leibler Divergence between Centered Multivariate generalized Gaussian Distributions*

Description

Computes the Kullback- Leibler divergence between two random vectors distributed according to multivariate generalized Gaussian distributions (MGGD) with zero means.

Usage

```
kldggd(Sigma1, beta1, Sigma2, beta2, eps = 1e-06)
```

Arguments

`Sigma1`	symmetric, positive-definite matrix. The dispersion matrix of the first distribution.
`beta1`	positive real number. The shape parameter of the first distribution.
`Sigma2`	symmetric, positive-definite matrix. The dispersion matrix of the second distribution.
`beta2`	positive real number. The shape parameter of the second distribution.
`eps`	numeric. Precision for the computation of the Lauricella D-hypergeometric function (see `lauricella`). Default: 1e-06.

Details

Given $\mathbf{X}_1$, a random vector of $\mathbb{R}^p$ ($p > 1$) distributed according to the MGGD with parameters $(\mathbf{0}, \Sigma_1, \beta_1)$ and $\mathbf{X}_2$, a random vector of $\mathbb{R}^p$ distributed according to the MGGD with parameters $(\mathbf{0}, \Sigma_2, \beta_2)$.

The Kullback-Leibler divergence between X_1 and X_2 is given by:

$$KL(\mathbf{X}_1||\mathbf{X}_2) = \ln\left(\frac{\beta_1|\Sigma_1|^{-1/2}\Gamma\left(\frac{p}{2\beta_2}\right)}{\beta_2|\Sigma_2|^{-1/2}\Gamma\left(\frac{p}{2\beta_1}\right)}\right) + \frac{p}{2}\left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right)\ln 2 - \frac{p}{2\beta_2} + 2^{\frac{\beta_2}{\beta_1}-1}\frac{\Gamma\left(\frac{\beta_2}{\beta_1}+\frac{p}{\beta_1}\right)}{\Gamma\left(\frac{p}{2\beta_1}\right)}\lambda_p^{\beta_2}$$

$$\times F_D^{(p-1)}\left(-\beta_1; \underbrace{\frac{1}{2},\ldots,\frac{1}{2}}_{p-1}; \frac{p}{2}; 1-\frac{\lambda_{p-1}}{\lambda_p},\ldots,1-\frac{\lambda_1}{\lambda_p}\right)$$

where $\lambda_1 < ... < \lambda_{p-1} < \lambda_p$ are the eigenvalues of the matrix $\Sigma_1\Sigma_2^{-1}$
and $F_D^{(p-1)}$ is the Lauricella D-hypergeometric function defined for p variables:

$$F_D^{(p)}\left(a; b_1, ..., b_p; g; x_1, ..., x_p\right) = \sum_{m_1 \geq 0} ... \sum_{m_p \geq 0} \frac{(a)_{m_1+...+m_p}(b_1)_{m_1} ... (b_p)_{m_p}}{(g)_{m_1+...+m_p}} \frac{x_1^{m_1}}{m_1!} ... \frac{x_p^{m_p}}{m_p!}$$

This computation uses the `lauricella` function.

When $p = 1$ (univariate case): let X_1, a random variable distributed according to the centered generalized Gaussian distribution with parameters $(0, \sigma_1, \beta_1)$ and X_2, a random variable distributed according to the generalized Gaussian distribution with parameters $(0, \sigma_2, \beta_2)$.

$$KL(X_1||X_2) = \ln\left(\frac{\frac{\beta_1}{\sqrt{\sigma_1}}\Gamma\left(\frac{1}{2\beta_2}\right)}{\frac{\beta_2}{\sqrt{\sigma_2}}\Gamma\left(\frac{1}{2\beta_1}\right)}\right) + \frac{1}{2}\left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right)\ln 2 - \frac{1}{2\beta_2} + 2^{\frac{\beta_2}{\beta_1}-1}\frac{\Gamma\left(\frac{\beta_2}{\beta_1}+\frac{1}{\beta_1}\right)}{\Gamma\left(\frac{1}{2\beta_1}\right)}\left(\frac{\sigma_1}{\sigma_2}\right)^{\beta_2}$$

Value

A numeric value: the Kullback-Leibler divergence between the two distributions, with two attributes `attr(, "epsilon")` (precision of the result of the Lauricella D-hypergeometric Function) and `attr(, "k")` (number of iterations) except when the distributions are univariate.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

N. Bouhlel, A. Dziri, Kullback-Leibler Divergence Between Multivariate Generalized Gaussian Distributions. IEEE Signal Processing Letters, vol. 26 no. 7, July 2019. doi:10.1109/LSP.2019.2915000

See Also

dmggd: probability density of a MGGD.

Examples

```
beta1 <- 0.74
beta2 <- 0.55
Sigma1 <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
Sigma2 <- matrix(c(1, 0.3, 0.2, 0.3, 0.5, 0.1, 0.2, 0.1, 0.7), nrow = 3)

# Kullback-Leibler divergence
kl12 <- kldggd(Sigma1, beta1, Sigma2, beta2)
kl21 <- kldggd(Sigma2, beta2, Sigma1, beta1)
print(kl12)
print(kl21)

# Distance (symmetrized Kullback-Leibler divergence)
kldist <- as.numeric(kl12) + as.numeric(kl21)
print(kldist)
```

---

kldstudent — *Kullback-Leibler Divergence between Centered Multivariate t Distributions*

Description

Computes the Kullback-Leibler divergence between two random vectors distributed according to multivariate t distributions (MTD) with zero location vector.

Usage

```
kldstudent(nu1, Sigma1, nu2, Sigma2, eps = 1e-06)
```

Arguments

`nu1`	numeric. The degrees of freedom of the first distribution.
`Sigma1`	symmetric, positive-definite matrix. The scatter matrix of the first distribution.
`nu2`	numeric. The degrees of freedom of the second distribution.
`Sigma2`	symmetric, positive-definite matrix. The scatter matrix of the second distribution.
`eps`	numeric. Precision for the computation of the partial derivative of the Lauricella D-hypergeometric function (see Details). Default: 1e-06.

Details

Given X_1, a random vector of $\mathbb{R}^p$ distributed according to the centered MTD with parameters $(\nu_1, 0, \Sigma_1)$ and X_2, a random vector of $\mathbb{R}^p$ distributed according to the MCD with parameters $(\nu_2, 0, \Sigma_2)$.

Let $\lambda_1, \dots, \lambda_p$ the eigenvalues of the square matrix $\Sigma_1 \Sigma_2^{-1}$ sorted in increasing order:

$$\lambda_1 < \dots < \lambda_{p-1} < \lambda_p$$

The Kullback-Leibler divergence of X_1 from X_2 is given by:

$$D_{KL}(\mathbf{X}_1\|\mathbf{X}_2) = \ln\left(\frac{\Gamma\left(\frac{\nu_1+p}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)\nu_2^{\frac{p}{2}}}{\Gamma\left(\frac{\nu_2+p}{2}\right)\Gamma\left(\frac{\nu_1}{2}\right)\nu_1^{\frac{p}{2}}}\right) + \frac{\nu_2-\nu_1}{2}\left[\psi\left(\frac{\nu_1+p}{2}\right) - \psi\left(\frac{\nu_1}{2}\right)\right] - \frac{1}{2}\sum_{i=1}^{p}\ln\lambda_i - \frac{\nu_2+p}{2}\times D$$

where ψ is the digamma function (see Special) and D is given by:

- If $\frac{\nu_1}{\nu_2}\lambda_1 > 1$,

$$D = \prod_{i=1}^{p}\left(\frac{\nu_2}{\nu_1}\frac{1}{\lambda_i}\right)^{\frac{1}{2}} \frac{\partial}{\partial a}\left\{F_D^{(p)}\left(\frac{\nu_1+p}{2}, \underbrace{\frac{1}{2},\ldots,\frac{1}{2}}_{p}; a+\frac{\nu_1+p}{2}; 1-\frac{\nu_2}{\nu_1}\frac{1}{\lambda_1},\ldots,1-\frac{\nu_2}{\nu_1}\frac{1}{\lambda_p}\right)\right\}\Big|_{a=0}$$

- If $\frac{\nu_1}{\nu_2}\lambda_p < 1$,

$$D = \frac{\partial}{\partial a}\left\{F_D^{(p)}\left(a, \underbrace{\frac{1}{2},\ldots,\frac{1}{2}}_{p}; a+\frac{\nu_1+p}{2}; 1-\frac{\nu_1}{\nu_2}\lambda_1,\ldots,1-\frac{\nu_1}{\nu_2}\lambda_p\right)\right\}\Big|_{a=0}$$

- If $\frac{\nu_1}{\nu_2}\lambda_1 < 1 < \frac{\nu 1}{\nu_2}\lambda_p$,

$$\begin{aligned} D &= -\ln\left(\frac{\nu_1}{\nu_2}\lambda_p\right) + \\ &\frac{\partial}{\partial a}\left\{F_D^{(p)}\left(a, \underbrace{\frac{1}{2},\ldots,\frac{1}{2}, a+\frac{\nu_1}{2}}_{p}; a+\frac{\nu_1+p}{2}; 1-\frac{\lambda_1}{\lambda_p},\ldots,1-\frac{\lambda_{p-1}}{\lambda_p}, 1-\frac{\nu_2}{\nu_1}\frac{1}{\lambda_p}\right)\right\}\Big|_{a=0}\end{aligned}$$

$F_D^{(p)}$ is the Lauricella D-hypergeometric function defined for p variables:

$$F_D^{(p)}\left(a; b_1,\ldots,b_p; g; x_1,\ldots,x_p\right) = \sum_{m_1\geq 0}\cdots\sum_{m_p\geq 0}\frac{(a)_{m_1+\cdots+m_p}(b_1)_{m_1}\ldots(b_p)_{m_p}}{(g)_{m_1+\cdots+m_p}}\frac{x_1^{m_1}}{m_1!}\cdots\frac{x_p^{m_p}}{m_p!}$$

Value

A numeric value: the Kullback-Leibler divergence between the two distributions, with two attributes `attr(, "epsilon")` (precision of the partial derivative of the Lauricella D-hypergeometric function,see Details) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

N. Bouhlel and D. Rousseau (2023), Exact Rényi and Kullback-Leibler Divergences Between Multivariate t-Distributions. IEEE Signal Processing Letters, vol. 30, pp. 1672-1676, October 2023. doi:10.1109/LSP.2023.3324594

Examples

```
nu1 <- 2
Sigma1 <- matrix(c(2, 1.2, 0.4, 1.2, 2, 0.6, 0.4, 0.6, 2), nrow = 3)
nu2 <- 4
Sigma2 <- matrix(c(1, 0.3, 0.1, 0.3, 1, 0.4, 0.1, 0.4, 1), nrow = 3)

kldstudent(nu1, Sigma1, nu2, Sigma2)
kldstudent(nu2, Sigma2, nu1, Sigma1)
```

---

lauricella — *Lauricella D-Hypergeometric Function*

Description

Computes the Lauricella D-hypergeometric function.

Usage

```
lauricella(a, b, g, x, eps = 1e-06)
```

Arguments

a	numeric.
b	numeric vector.
g	numeric.
x	numeric vector. x must have the same length as b.
eps	numeric. Precision for the nested sums (default 1e-06).

Details

If n is the length of the b and x vectors, the Lauricella D-hypergeometric function is given by:

$$F_D^{(n)}\left(a, b_1, ..., b_n, g; x_1, ..., x_n\right) = \sum_{m_1 \geq 0} ... \sum_{m_n \geq 0} \frac{(a)_{m_1+...+m_n}(b_1)_{m_1} ... (b_n)_{m_n}}{(g)_{m_1+...+m_n}} \frac{x_1^{m_1}}{m_1!} ... \frac{x_n^{m_n}}{m_n!}$$

where $(x)_p$ is the Pochhammer symbol (see pochhammer).

If $|x_i| < 1, i = 1, \ldots, n$, this sum converges. Otherwise there is an error.

The eps argument gives the required precision for its computation. It is the `attr(, "epsilon")` attribute of the returned value.

Value

A numeric value: the value of the Lauricella function, with two attributes `attr(, "epsilon")` (precision of the result) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

N. Bouhlel, A. Dziri, Kullback-Leibler Divergence Between Multivariate Generalized Gaussian Distributions. IEEE Signal Processing Letters, vol. 26 no. 7, July 2019. doi:10.1109/LSP.2019.2915000

N. Bouhlel and D. Rousseau (2023), Exact Rényi and Kullback-Leibler Divergences Between Multivariate t-Distributions. IEEE Signal Processing Letters, vol. 30, pp. 1672-1676, October 2023. doi:10.1109/LSP.2023.3324594

`lnpochhammer` *Logarithm of the Pochhammer Symbol*

Description

Computes the logarithm of the Pochhammer symbol.

Usage

```
lnpochhammer(x, n)
```

Arguments

`x`	numeric.
`n`	positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1)$$

So, if $n > 0$:

$$log\left((x)_n\right) = log(x) + log(x+1) + ... + log(x+n-1)$$

If $n = 0$, $log\left((x)_n\right) = log(1) = 0$

Value

Numeric value. The logarithm of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

See Also

pochhammer, lauricella

Examples

```
lnpochhammer(2, 0)
lnpochhammer(2, 1)
lnpochhammer(2, 3)
```

---

plotmvd *Plot a Bivariate Density*

Description

Plots the probability density of a multivariate distribution with 2 variables:

- generalized Gaussian distribution (MGGD) with mean vector mu, dispersion matrix Sigma and shape parameter beta
- Cauchy distribution (MCD) with location parameter mu and scatter matrix Sigma
- t distribution (MTD) with location parameter mu and scatter matrix Sigma

This function uses the plot3d.function function.

Usage

```
plotmvd(mu, Sigma, beta = NULL, nu = NULL,
               distribution = c("mggd", "mcd", "mtd"),
               xlim = c(mu[1] + c(-10, 10)*Sigma[1, 1]),
               ylim = c(mu[2] + c(-10, 10)*Sigma[2, 2]), n = 101,
               xvals = NULL, yvals = NULL, xlab = "x", ylab = "y",
               zlab = "f(x,y)", col = "gray", tol = 1e-6, ...)
```

Arguments

mu	length 2 numeric vector.
Sigma	symmetric, positive-definite square matrix of order 2.
beta	numeric. If distribution = "mggd", the shape parameter of the MGGD. NULL if dist is "mcd" or "mtd".
nu	numeric. If distribution = "mtd", the degrees of freedom of the MTD. NULL if distribution is "mggd" or "mcd".
distribution	the probability distribution. It can be "mggd" (multivariate generalized Gaussian distribution) "mcd" (multivariate Cauchy) or "mtd" (multivariate t).
xlim, ylim	x-and y- limits.
n	A one or two element vector giving the number of steps in the x and y grid, passed to plot3d.function.

xvals, yvals	The values at which to evaluate x and y. If used, xlim and/or ylim are ignored.
xlab, ylab, zlab	The axis labels.
col	The color to use for the plot. See plot3d.function.
tol	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma, for the estimation of the density. See dmggd, dmcd or dmtd.
...	Additional arguments to pass to plot3d.function.

Value

Returns invisibly the probability density function.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

S. Kotz and Saralees Nadarajah (2004), Multivariate t Distributions and Their Applications, Cambridge University Press.

See Also

contourmvd: contour plot of a bivariate generalised Gaussian, Cauchy or t density.

dmggd: Probability density of a multivariate generalised Gaussian distribution.

dmcd: Probability density of a multivariate Cauchy distribution.

dmtd: Probability density of a multivariate t distribution.

Examples

```
mu <- c(1, 4)
Sigma <- matrix(c(0.8, 0.2, 0.2, 0.2), nrow = 2)

# Bivariate generalised Gaussian distribution
beta <- 0.74
plotmvd(mu, Sigma, beta = beta, distribution = "mggd")


# Bivariate Cauchy distribution
plotmvd(mu, Sigma, distribution = "mcd")

# Bivariate t distribution
nu <- 2
plotmvd(mu, Sigma, nu = nu, distribution = "mtd")
```

`pochhammer` *Pochhammer Symbol*

Description

Computes the Pochhammer symbol.

Usage

```
pochhammer(x, n)
```

Arguments

`x`	numeric.
`n`	positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1)$$

Value

Numeric value. The value of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

See Also

`lauricella`

Examples

```
pochhammer(2, 0)
pochhammer(2, 1)
pochhammer(2, 3)
```

rmcd *Simulate from a Multivariate Cauchy Distribution*

Description

Produces one or more samples from the multivariate (p variables) Cauchy distribution (MCD) with location parameter `mu` and scatter matrix `Sigma`.

Usage

```
rmcd(n, mu, Sigma, tol = 1e-6)
```

Arguments

`n`	integer. Number of observations.
`mu`	length p numeric vector. The location parameter.
`Sigma`	symmetric, positive-definite square matrix of order p. The scatter matrix.
`tol`	tolerance for numerical lack of positive-definiteness in Sigma (for `mvrnorm`, see Details).

Details

A sample from a MCD with parameters $\boldsymbol{\mu}$ and Σ can be generated using:

$$\mathbf{X} = \boldsymbol{\mu} + \frac{\mathbf{Y}}{\sqrt{u}}$$

where $\mathbf{Y}$ is a random vector distributed among a centered Gaussian density with covariance matrix Σ (generated using `mvrnorm`) and u is distributed among a Chi-squared distribution with 1 degree of freedom.

Value

A matrix with p columns and n rows.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

S. Kotz and Saralees Nadarajah (2004), Multivariate t Distributions and Their Applications, Cambridge University Press.

See Also

`dmcd`: probability density of a MCD.

`estparmcd`: estimation of the parameters of a MCD.

Examples

```
mu <- c(0, 1, 4)
sigma <- matrix(c(1, 0.6, 0.2, 0.6, 1, 0.3, 0.2, 0.3, 1), nrow = 3)
x <- rmcd(100, mu, sigma)
x
apply(x, 2, median)
```

---

rmggd — *Simulate from a Multivariate Generalized Gaussian Distribution*

Description

Produces one or more samples from a multivariate (p variables) generalized Gaussian distribution (MGGD).

Usage

```
rmggd(n = 1 , mu, Sigma, beta, tol = 1e-6)
```

Arguments

`n`	integer. Number of observations.
`mu`	length p numeric vector. The mean vector.
`Sigma`	symmetric, positive-definite square matrix of order p. The dispersion matrix.
`beta`	positive real number. The shape of the distribution.
`tol`	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma.

Details

A sample from a centered MGGD with dispersion matrix Σ and shape parameter β can be generated using:

$$X = \tau \, \Sigma^{1/2} \, U$$

where U is a random vector uniformly distributed on the unit sphere and τ is such that $\tau^{2\beta}$ is generated from a distribution Gamma with shape parameter $\frac{p}{2\beta}$ and scale parameter 2.

Value

A matrix with p columns and `n` rows.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

See Also

`dmggd`: probability density of a MGGD..

`estparmggd`: estimation of the parameters of a MGGD.

Examples

```
mu <- c(0, 0, 0)
Sigma <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
beta <- 0.74
rmggd(100, mu, Sigma, beta)
```

`rmtd` *Simulate from a Multivariate t Distribution*

Description

Produces one or more samples from the multivariate (p variables) t distribution (MTD) with degrees of freedom `nu`, mean vector `mu` and correlation matrix `Sigma`.

Usage

```
rmtd(n, nu, mu, Sigma, tol = 1e-6)
```

Arguments

`n`	integer. Number of observations.
`nu`	numeric. The degrees of freedom.
`mu`	length p numeric vector. The mean vector
`Sigma`	symmetric, positive-definite square matrix of order p. The correlation matrix.
`tol`	tolerance for numerical lack of positive-definiteness in Sigma (for `mvrnorm`, see Details).

Details

A sample from a MTD with parameters ν, $\boldsymbol{\mu}$ and Σ can be generated using:

$$\mathbf{X} = \boldsymbol{\mu} + \mathbf{Y}\sqrt{\frac{\nu}{u}}$$

where Y is a random vector distributed among a centered Gaussian density with covariance matrix Σ (generated using `mvrnorm`) and u is distributed among a Chi-squared distribution with ν degrees of freedom.

Value

A matrix with p columns and n rows.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

S. Kotz and Saralees Nadarajah (2004), Multivariate t Distributions and Their Applications, Cambridge University Press.

See Also

dmtd: probability density of a MTD.

estparmtd: estimation of the parameters of a MTD.

Examples

```
nu <- 3
mu <- c(0, 1, 4)
Sigma <- matrix(c(1, 0.6, 0.2, 0.6, 1, 0.3, 0.2, 0.3, 1), nrow = 3)
x <- rmtd(10000, nu, mu, Sigma)
head(x)
dim(x)
mu; colMeans(x)
nu/(nu-2)*Sigma; var(x)
```

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