Optimizing Costs of Fish Stocking

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1 Introduction

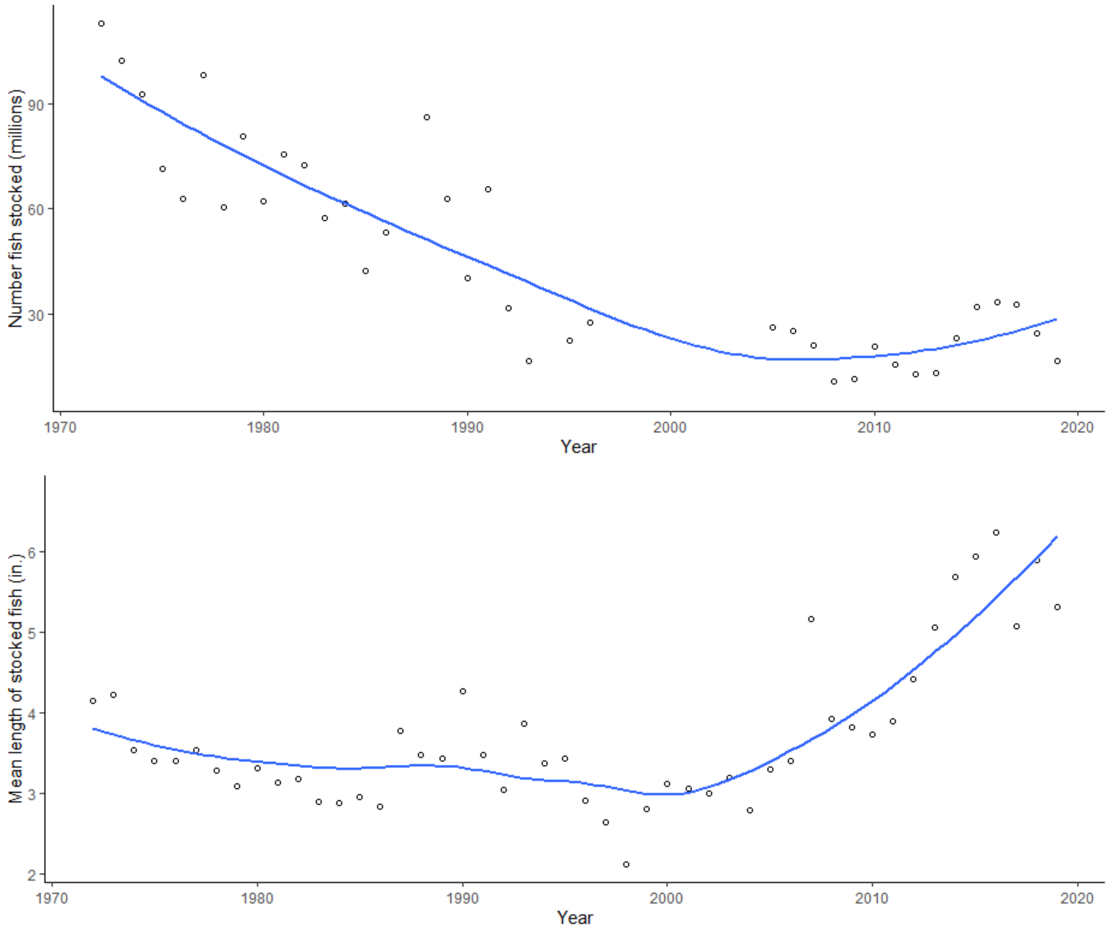
Initial propagation efforts by the Wisconsin DNR focused on stocking many small fish into waterbodies throughout the state. As time has gone by stocking strategies have shifted from this to one that inputs fewer, but larger fish into systems (Fig. 1). This is done presumably with the intent that larger fish are less susceptible to natural mortality resulting in more fish recruiting to the minimum legal limit (MLL). This is a good strategy, but, as is shown in this model, may not be the most cost effective.

This model uses knowledge about a fish species' growth and mortality to determine how many fish should be stocked to achieve a desired number of fish recruiting into the fishery at MLL. From this the costs of stocking can be used to determine the most cost effective timeframe at which to stock a particular species of fish into a waterbody.

2 Determining Time to Recruitment into the Fishery

With knowledge about a fishes growth, one can determine the time that it takes for fish to grow to the MLL. A basic inversion of the von Bertalanffy growth function (VBGF) will predict the time that it takes for a fish to reach MLL. Since daily values for hatchery costs will be used in this model the time to MLL will be given in days. A straightforward and relatively close transformation from yearly VBGF parameters (i.e. growth across age in years) to daily parameter values can be closely approximated by leaving all yearly VBGF parameters the same except k which gets divided by 365.

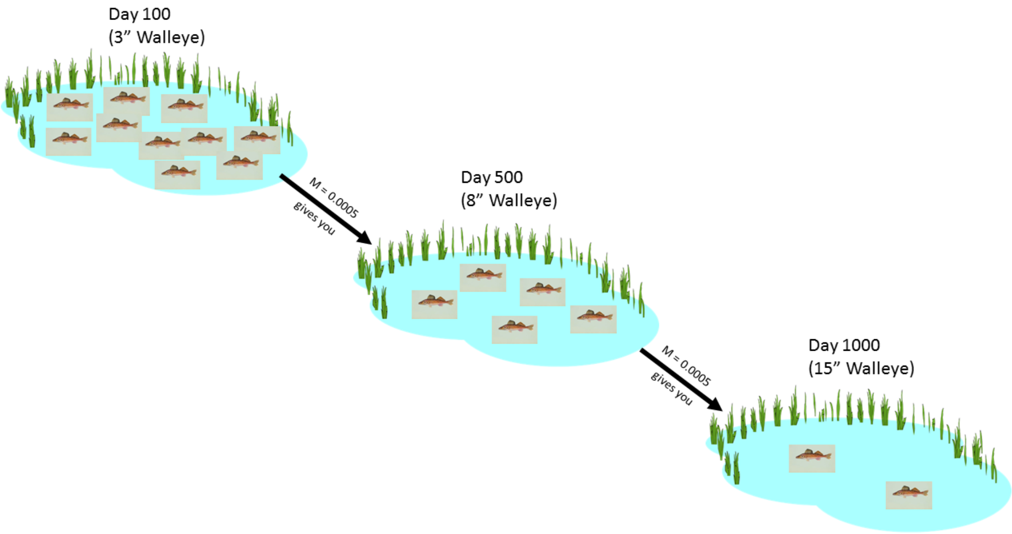
Figure 1: Total number (top) and mean length (bottom) of walleyes stocked into Wisconsin waterbodies across time. Data retreived from FMDB.



3 Using natural mortality to gauge stocking number for desired number of recruits

The number of fish at a future time interval can be determined by the number of fish at the present time multiplied by the exponent of natural mortality. This equation can be reversed and used to backcast how many fish should be stocked at various different times to achieve a desired number of fish recruiting into the fishery (Fig. 2).

Figure 2: Using natural mortality one can determine how many fish should be stocked at various time intervals to achieve a certain number recruiting into the fishery at a given minimum length limit.



4 Adding in Costs

If we know the average age at which fish are recruiting into the fishery (i.e. the MLL – using the inverse VBGF), and we know how many fish must be stocked into the system on a particular day to achieve a certain number of recruits at MLL (using assumptions about mortality) we can use daily hatchery costs to estimate the total cost of raising x number of fish until the day at which we choose to stock those fish. This will provide the cost-per-fish for any given day that is chosen for stocking, and a curve can be plotted for these values. All that's left to do is to find the minimum cost-per-fish across all days from day 0 (hatch day) until the day at which average fish recruit at the MLL. This minimum will provide the most cost-effective strategy for stocking fish into a system in order to supply a certain number of fish recruiting to the MLL.

Appendix - Model Details

If growth of a fish species is known, the inverse of the growth function can be used to determine the mean time that it takes for fish to grow to a particular length. Let L_R be the length at recruitment into a fishery. For angling purposes this is likely best thought of as the minimum legal length (MLL). For the purposes of illustrating this model the von Bertalanffy growth function (VBGF) is used:

$$L_a = L_\infty(1 - \exp(-k(a - t0))) \tag{1}$$

where L_a is the mean length at age a, and L, k, and t_0 are parameters that define the curve across a. This function can be rearranged to calculate the time, t_R, that it takes to grow until length L_R as follows:

$$t_R = \frac{\log\left(\frac{\frac{L_R}{L_\infty} - 1}{-1}\right)}{-k} + t_0 \tag{2}$$

Using t_R as a reference point for when fish recruit into the system one can use assumptions about mortality to determine how many fish need to be stocked on any given day to provide the resulting desired number of recruits (N_R) at time t_R. This can be accomplished using a basic age-structured population that decreases across time according to some natural mortality function. Let M be a constant mortality across time. The population decreases at each time interval according to the following:

$$\begin{aligned} N_1 &= N_0 e^{-M} \\ N_2 &= N_1 e^{-M} \\ &\ldots \\ N_R &= N_{R-1} e^{-M} \end{aligned}$$

or, written another way,

$$\begin{aligned} N_1 &= N_0 e^{-M} \\ N_2 &= N_0 e^{-M} e^{-M} \\ &\ldots \\ N_R &= N_0 \prod_{t=1}^{R} e^{-M} \\ &= N_0 e^{\sum_{t=1}^{R} -M} \end{aligned}$$

Since constant mortality is a function across time t the above can be generalized to:

$$N_R = N_0 e^{\int_{t=0}^{t=t_R} f(t)} \tag{3}$$

where $f(t)$ is some mortality function across time. The number of fish N_0 can be assumed to be equal to the number of fish stocked (N_S) since this number is known. Eqn. 3 can then be rearranged to get N_S, which is the number of fish that need to be stocked to result in the number of recruits (N_R) at time t_R given the mortality function.

$$N_S = \frac{1}{\displaystyle\int_{t=0}^{t=t_R} f(t)} \cdot N_R$$

From here the daily costs of rearing N_S number of fish in a hatchery until time at stocking (t_S) can be accumulated. This can be done either with a total cost or by integrating a daily cost equation across time:

$$\mathrm{T}_S = \int_{t=0}^{s} C(t) \tag{4}$$

where T_S is the total cost to raise N_S fish to time t. From here the cost-per-fish at stocking time S is then:

$$\mathrm{CPF_S} = \frac{T_S}{N_S} \tag{5}$$

where $\mathrm{CPF_S}$ is the cost-per-fish at stocking time S. This provides a curve when plotted across time, which can subsequently be minimimzed to obtain the lowest value for CPF, which is equal to the most cost-effective time to stock a species given growth, mortality, and rearing costs.