Formelsammlung

zur Vorlesung Statistik I + II

Humboldt-Universität zu Berlin

Wirtschaftswissenschaftliche Fakultät
Lehrstuhl für Statistik

8. April 2022

Inhaltsverzeichnis

1 Univariate Statistik

1.1 Verteilung von Variablen

1.1.1 Verteilung klassierter Variablen

Anzahl der Beobachtungen	n	
Anzahl der Klassen	k	$j = 1, \ldots, k$
Untere/obere Klassengrenze	x_j^u	x_j^o
mit	$x_j^o = x_{j+1}^u$	$x_j^u < x \leq x_j^o$
Klassenbreite, Klassenmitte	$\triangle x_j = x_j^o - x_j^u$	$x_j^m = \frac{1}{2}(x_j^u + x_j^o)$

Empirische Häufigkeitsverteilung klassierter Variablen

Absolute Klassenhäufigkeit:

$$h(x_j) \quad = \quad h(x_j^u < X \leq x_j^o) = h_j = \sum_{i=1}^{n} I(x_j^u < x_i \leq x_j^o)$$

relative Klassenhäufigkeit:

$$f(x_j) = f(x_j^u < X \leq x_j^o) = \frac{h(x_j)}{n}$$

Häufigkeitsdichte:

$$f_K(x_j) = \frac{f(x_j)}{x_j^o - x_j^u} \text{ für } x_j^u < X \leq x_j^o$$

Empirische Verteilungsfunktion klassierter Variablen

$$F(x) = \begin{cases} 0 & \text{für } \; x \leq x_1^u \\ \sum\limits_{i=1}^{j-1} f(x_i) + \dfrac{x - x_j^u}{x_j^o - x_j^u} \cdot f(x_j) & \text{für } \; x_j^u < x \leq x_j^o \\ 1 & \text{für } \; x_k^o < x \end{cases}$$

Interpolation von $F(x)$

$$F(x) = F(x_j^u) + \frac{x - x_j^u}{x_j^o - x_j^u} \cdot f(x_j)$$

1.1.2 Verteilung unklassierter Variablen

Empirische Häufigkeitsverteilung

Anzahl der Beobachtungen	n
absolute Häufigkeit	$h(x_j) = h(X = x_j) = h_j = \sum_{i=1}^{n} I(x_i = x_j)$
relative Häufigkeit	$f(x_j) = \frac{h(x_j)}{n}$

Empirische Verteilungsfunktion

$$F(x) = \begin{cases} 0 & \text{für} \quad x < x_1 \\ \sum\limits_{i=1}^{j} f(x_i) & \text{für} \quad x_j \leq x < x_{j+1} \\ 1 & \text{für} \quad x_k \leq x \end{cases}$$

Anzahl der Merkmalsausprägungen k

absolute Summenhäufigkeit $H(x_j) = \sum\limits_{i=1}^{j} h(x_i) \quad \text{für} \quad j = 1, \ldots, k$

1.2 Parameter von Variablen

1.2.1 Lageparameter

Arithmetisches Mittel

unklassierte Variablen $\quad \overline{x} = \frac{1}{n} \cdot \sum\limits_{i=1}^{n} x_i$

diskrete Variablen $\quad \overline{x} = \frac{1}{n} \cdot \sum\limits_{j=1}^{k} x_j \cdot h(x_j) = \sum\limits_{j=1}^{k} x_j \cdot f(x_j)$

klassierte Variablen $\quad \overline{x} = \frac{1}{n} \cdot \sum\limits_{j=1}^{k} x_j^m \cdot h(x_j) = \sum\limits_{j=1}^{k} x_j^m \cdot f(x_j)$

gewogenes $\quad \overline{x} = \sum\limits_{i=1}^{n} x_i \cdot g_i \Big/ \sum\limits_{i=1}^{n} g_i$

gepooltes $\quad \overline{x} = \sum\limits_{\ell=1}^{r} \frac{n_\ell}{n} \, \overline{x}_\ell$

n_ℓ Beobachtungen und $\overline{x}_\ell$ Mittelwert in Gruppe ℓ

Modus

Nichtklassierte Variablen:

$$x_D = \left\{ x_j \mid h_j = \max_{x_k} h_k \text{ bzw. } f_j = \max_{x_k} f(x_k) \right\}$$

Klassierte Variablen:

$$x_D = x_j^u + \frac{f_K(x_j) - f_K(x_{j-1})}{2 \cdot f_K(x_j) - f_K(x_{j-1}) - f_K(x_{j+1})} \cdot (x_j^o - x_j^u)$$

Median

nichtklassierte Variablen $\quad x_{0,5} = x_{\left(\frac{n+1}{2}\right)} \quad$ falls n ungerade

$\quad x_{0,5} = \frac{1}{2} \cdot \left\{ x_{\left(\frac{n}{2}\right)} + x_{\left(\frac{n}{2}+1\right)} \right\} \quad$ falls n gerade

klassierte Variablen $\quad x_{0,5} = x_j^u + \frac{0,5 - F(x_j^u)}{f(x_j)} \cdot (x_j^o - x_j^u)$

p - Quantile

Nichtklassierte Variablen:

$x_p = x_{(k)} \quad$ falls $n \cdot p \notin \mathbb{Z}$ und $k \in \mathbb{Z}$ die auf $n \cdot p$ folgende ganze Zahl

$x_p = \frac{1}{2} \cdot \left\{ x_{(k)} + x_{(k+1)} \right\} \quad$ falls $n \cdot p \in \mathbb{Z}$, dann $k = n \cdot p$

Klassierte Variablen:

$$x_p = x_j^u + \frac{p - F(x_j^u)}{f(x_j)} \cdot (x_j^o - x_j^u) \quad \text{für } 0 < p \leq 1$$

Harmonisches Mittel

einfaches $\quad \overline{x}_H = \frac{n}{\sum\limits_{i=1}^{n} \frac{1}{x_i}}$

gewogenes $\quad \overline{x}_H = \frac{\sum\limits_{j=1}^{k} g_j}{\sum\limits_{j=1}^{k} \frac{g_j}{x_j}} \quad$ mit $x_j = \frac{g_j}{h_j}, \quad j = 1, \ldots, k$

1.2.2 Streuungsparameter

Spannweite

$$R = x_{max} - x_{min} = x_{(n)} - x_{(1)}$$

Quartilsabstand, Interquartilsabstand

$$QA = x_{0,75} - x_{0,25}$$

Lineares Streuungsmaß (Mittlere absolute Abweichung)

$$d = \frac{1}{n} \cdot \sum_{j=1}^{n} |x_i - c|, \text{ mit } c = x_{0,5} \text{ oder } c = \overline{x}$$

Varianz einer empirischen Häufigkeitsverteilung

$$\begin{aligned} s^2 &= \frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \overline{x})^2 = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i^2 - \overline{x}^2 \\ &= \frac{1}{n} \cdot \sum_{j=1}^{k} (x_j - \overline{x})^2 \cdot h(x_j) = \sum_{j=1}^{k} (x_j - \overline{x})^2 \cdot f(x_j) \end{aligned}$$

Standardabweichung einer empirischen Häufigkeitsverteilung

$$s = \sqrt{s^2}$$

Gepoolte Varianz

$$s^2 = \sum_{\ell=1}^{r} \frac{n_\ell}{n} \cdot s_\ell^2 + \sum_{\ell=1}^{r} \frac{n_\ell}{n} \cdot (\overline{x}_\ell - \overline{x})^2$$

mit n_ℓ Beobachtungen, $\overline{x}_\ell$ Mittelwert und s_ℓ^2 die Varianz in Gruppe ℓ

Variations- und Quartilsdispersionskoeffizient

$$v = \frac{s}{\overline{x}} \text{ für } \overline{x} > 0 \qquad q = \frac{QA}{x_{0,5}} \text{ für } x_{0,5} > 0$$

2 Bivariate Statistik

2.1 Verteilung von Variablen

Gemeinsame Verteilung

Absolute Häufigkeit $h(x_i, y_j) = h_{ij} = \sum_{k=1}^{m} \sum_{l=1}^{r} I\left((x_k, y_l) = (x_i, y_j)\right)$

Relative Häufigkeit $f(x_i, y_j) = f_{ij} = \frac{h_{ij}}{n}$

Verteilungsfunktion $F(x, y) = \sum_{x_i \leq x} \sum_{y_j \leq y} f(x_i, y_j)$

Randverteilungen

Absolute Häufigkeit für X $h_{i\bullet} = \sum_{j=1}^{r} h_{ij} \quad i = 1, \ldots, m$

Relative Häufigkeit für X $f_{i\bullet} = \sum_{j=1}^{r} f_{ij} \quad i = 1, \ldots, m$

Absolute Häufigkeit für Y $h_{\bullet j} = \sum_{i=1}^{m} h_{ij} \quad j = 1, \ldots, r$

Relative Häufigkeit für Y $f_{\bullet j} = \sum_{i=1}^{m} f_{ij} \quad j = 1, \ldots, r$

Bedingte Verteilungen

Relative Häufigkeit bedingt auf Y $f(x_i|y_j) = \frac{f_{ij}}{f_{\bullet j}} = \frac{h_{ij}}{h_{\bullet j}}$

Relative Häufigkeit bedingt auf X $f(y_j|x_i) = \frac{f_{ij}}{f_{i\bullet}} = \frac{h_{ij}}{h_{i\bullet}}$

2.2 Maßzahlen für den Zusammenhang zweier Variablen

2.2.1 Empirische Kovarianz

$$s_{xy} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \overline{x})(y_j - \overline{y})$$

2.2.2 Bravais-Pearson-Korrelationskoeffizient

$$r_{xy} = r_{yx} = \frac{s_{xy}}{s_x \cdot s_y} = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2 \cdot \sum_{i=1}^{n}(y_i - \overline{y})^2}} \quad \text{mit } -1 \leq r_{xy} \leq +1$$

$$= \frac{n \cdot \sum_{i=1}^{n} x_i \cdot y_i - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i}{\sqrt{\left(n \cdot \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right)\left(n \cdot \sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right)}}$$

2.2.3 Spearman'scher Rangkorrelationskoeffizient

$$r_s = 1 - \frac{6 \cdot \sum_{i=1}^{n} d_i^2}{n \cdot (n^2 - 1)} \text{ mit } d_i = \text{Rang}(x_i) - \text{Rang}(y_i) \text{ und } -1 \leq r_s \leq +1$$

2.2.4 Kendall'scher Rangkorrelationskoeffizient

$$\tau = \frac{P - Q}{P + Q} \quad \text{mit } -1 \leq \tau \leq +1$$

P die Anzahl der Beobachtungspaare mit $x_i < x_j$ und $y_i < y_j$ sowie
Q die Anzahl der Beobachtungspaare mit $x_i < x_j$ und $y_i > y_j$

2.2.5 Quadratische Kontingenz

$$K^2 = \sum_{i=1}^{m}\sum_{j=1}^{r} \frac{(h_{ij} - \hat{e}_{ij})^2}{\hat{e}_{ij}} = n \cdot \left(-1 + \sum_{i=1}^{m}\sum_{j=1}^{r} \frac{h_{ij}^2}{h_{i\bullet} \cdot h_{\bullet j}}\right)$$

$$= n \cdot \sum_{i=1}^{m}\sum_{j=1}^{r} \frac{(f_{ij} - f_{i\bullet} \cdot f_{\bullet j})^2}{f_{i\bullet} \cdot f_{\bullet j}}$$

mit $\hat{e}_{ij} = \frac{1}{n} \cdot h_{i\bullet} \cdot h_{\bullet j}$ (erwartete Häufigkeit unter Unabhängigkeit).

2.2.6 Kontingenzkoeffizient und korrigierter Kontingenzkoeffizient

$$C = \sqrt{\frac{K^2}{n + K^2}}; \qquad C_{korr} = C \cdot \sqrt{\frac{C^*}{C^* - 1}} \text{ mit } C^* = \min(\text{Anzahl Zeilen}, \text{Anzahl Spalten})$$

3 Lineare Regression

3.1 Regressiongerade

$$\hat{y}_i = b_0 + b_1 \cdot x_i + \epsilon_i$$

$$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n}(y_i - b_0 - b_1 \cdot x_i)^2 \to \text{ minimal}$$

3.2 Regressionskoeffizienten

3.2.1 Steigung

$$b_1 = \frac{\sum_{i=1}^{n}(x_i - \overline{x}) \cdot (y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

$$= \frac{n \cdot \sum_{i=1}^{n} x_i \cdot y_i - \left(\sum_{i=1}^{n} x_i\right) \cdot \left(\sum_{i=1}^{n} y_i\right)}{n \cdot \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$= \frac{s_{xy}}{s_x^2} = r_{xy} \cdot \frac{s_y}{s_x}$$

3.2.2 Achsenabschnitt

$$b_0 = \frac{\sum_{i=1}^{n} y_i \cdot \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} x_i \cdot y_i}{n \cdot \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$= \overline{y} - b_1 \cdot \overline{x}$$

3.3 Bestimmtheitsmaß und Korrelation

$$R_{yx}^2 = R_{xy}^2 = \frac{s_{yx}^2}{s_y^2 \cdot s_x^2} = r_{yx}^2 = \frac{\sum_{i=1}^{n}(\hat{y}_i - \overline{y})^2}{\sum_{i=1}^{n}(y_i - \overline{y})^2} = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \overline{y})^2}$$

$$= \frac{\left[\sum_{i=1}^{n}(y_i - \bar{y}) \cdot (x_i - \bar{x})\right]^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2 \cdot \sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$= \frac{\left(n \cdot \sum_{i=1}^{n} x_i \cdot y_i - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i\right)^2}{\left[n \cdot \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right]\left[n \cdot \sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right]}$$

4 Zeitreihenanalyse

4.1 Geometrisches Mittel

Geometrisches Mittel $\overline{x}_G = \sqrt[n]{x_1 \cdot x_2 \cdot \ldots \cdot x_n}$

Mittleres Wachstum $i_G = \sqrt[n]{\frac{x_1}{x_0} \cdot \frac{x_2}{x_1} \cdot \ldots \cdot \frac{x_n}{x_{n-1}}} = \sqrt[n]{\frac{x_n}{x_0}}$

4.2 Trendbestimmung

4.2.1 Gleitender Durchschnitt

Ordnung	x_t^* mit $t = k+1, \ldots, T-k$
ungerade	$\frac{1}{2k+1} \cdot \sum_{i=t-k}^{t+k} x_i$
gerade	$\frac{1}{2k} \cdot \left[\frac{1}{2} \cdot x_{t-k} + \frac{1}{2} \cdot x_{t+k} + \sum_{i=t-(k-1)}^{t+(k-1)} x_i\right]$

4.2.2 Lineare Trendfunktion

Trendfunktion $\hat{x}_t = a + b \cdot t$

Schätzwerte

$$a = \frac{\sum_{t=1}^{T} x_t \cdot \sum_{t=1}^{T} t^2 - \sum_{t=1}^{T} t \cdot \sum_{t=1}^{T} x_t \cdot t}{T \cdot \sum_{t=1}^{T} t^2 - \left(\sum_{t=1}^{T} t\right)^2}$$

$$b = \frac{T \cdot \sum_{t=1}^{T} x_t \cdot t - \sum_{t=1}^{T} x_t \sum_{t=1}^{T} t}{T \cdot \sum_{t=1}^{T} t^2 - \left(\sum_{t=1}^{T} t\right)^2}$$

4.2.3 Exponentialtrend

$$\text{Trendfunktion} \quad \hat{x}_t = a \cdot b^t \Longleftrightarrow \log \hat{x}_t = \log a + t \cdot \log b$$

$$\text{Schätzwerte} \quad \log a = \frac{\sum_{t=1}^{T} \log x_t \cdot \sum_{t=1}^{T} t^2 - \sum_{t=1}^{T} t \cdot \sum_{t=1}^{T} t \cdot \log x_t}{T \cdot \sum_{t=1}^{T} t^2 - \left(\sum_{t=1}^{T} t \right)^2}$$

$$\log b = \frac{T \cdot \sum_{t=1}^{T} t \cdot \log x_t - \sum_{t=1}^{T} \log x_t \cdot \sum_{t=1}^{T} t}{T \cdot \sum_{t=1}^{T} t^2 - \left(\sum_{t=1}^{T} t \right)^2}$$

4.3 Periodische Schwankungen

Zeitreihenmodell	Additiv	Multiplikativ
$s_{i,j} =$	$x_{i,j} - \hat{x}_{i,j}$	$\frac{x_{i,j}}{\hat{x}_{i,j}}$
$\overline{s}_j =$	$\frac{1}{P} \cdot \sum_{i=1}^{P} s_{i,j}$	$\frac{1}{P} \cdot \sum_{i=1}^{P} s_{i,j}$
$\hat{x}_{i,j}^{ZRM} =$	$\hat{x}_{i,j} + \overline{s}_j$	$\hat{x}_{i,j} \cdot \overline{s}_j$

4.4 Gütemaße

4.4.1 Mittlere quadratische Streuung (Standardabweichung)

$$s_{ZRM} = \sqrt{\frac{1}{T} \sum_{i=1}^{P} \sum_{j=1}^{k} (x_{i,j} - \hat{x}_{i,j}^{ZRM})^2}$$

4.4.2 Variationskoeffizient

$$v = \frac{s_{ZRM}}{\overline{x}}$$

4.4.3 Bestimmtheitsmaß

$$R^2 = 1 - \frac{s_{ZRM}^2}{s_x^2} \text{ mit } s_x^2 = \frac{1}{T} \sum_{i=1}^{P} \sum_{j=1}^{k} (x_{i,j} - \overline{x})^2, \quad 0 \leq \frac{s_{ZRM}^2}{s_x^2} \leq 1$$

5 Indexzahlen

Anzahl der Güter im Warenkorb: n		
	Basiszeitraum $t = 0$	Berichtszeitraum t
Preis des Gutes i	$p_0(i)$	$p_t(i)$
Menge des Gutes i	$q_0(i)$	$q_t(i)$
Wert des Gutes i	$v_0(i) = p_0(i) \cdot q_0(i)$	$v_t(i) = p_t(i) \cdot q_t(i)$

5.1 Messzahlen

Preismesszahl für das Gut i: $\frac{p_t(i)}{p_0(i)}$

Mengenmesszahl für das Gut i: $\frac{q_t(i)}{q_0(i)}$

Wertmesszahl für das Gut i: $\frac{v_t(i)}{v_0(i)} = \frac{p_t(i)}{p_0(i)} \frac{q_t(i)}{q_0(i)}$

5.2 Indices

5.2.1 Nach Laspeyres

Preisindex: $$I_{La;0,t}^{p} = \sum_{i=1}^{n} \frac{p_t(i)}{p_0(i)} \cdot \frac{p_0(i) q_0(i)}{\sum_{j=1}^{n} p_0(j) q_0(j)} = \frac{\sum_{i=1}^{n} p_t(i) q_0(i)}{\sum_{i=1}^{n} p_0(i) q_0(i)}$$

Mengenindex: $$I_{La;0,t}^{q} = \sum_{i=1}^{n} \frac{q_t(i)}{q_0(i)} \cdot \frac{p_0(i) q_0(i)}{\sum_{j=1}^{n} p_0(j) q_0(j)} = \frac{\sum_{i=1}^{n} q_t(i) p_0(i)}{\sum_{i=1}^{n} q_0(i) p_0(i)}$$

Wertindex: $$I_{La;0,t}^{v} = \sum_{i=1}^{n} \frac{v_t(i)}{v_0(i)} \cdot \frac{v_0(i)}{\sum_{j=1}^{n} v_0(j)} = \frac{\sum_{i=1}^{n} p_t(i) q_t(i)}{\sum_{i=1}^{n} p_0(i) q_0(i)}$$

5.2.2 Nach Paasche

Preisindex: $$I^p_{Pa;0,t} = \frac{1}{\sum\limits_{i=1}^{n} \frac{1}{\frac{p_t(i)}{p_0(i)}} \cdot \frac{p_t(i)q_t(i)}{\sum_{j=1}^{n} p_t(j)q_t(j)}} = \frac{\sum\limits_{i=1}^{n} p_t(i)q_t(i)}{\sum\limits_{i=1}^{n} p_0(i)q_t(i)}$$

Mengenindex: $$I^q_{Pa;0,t} = \frac{1}{\sum\limits_{i=1}^{n} \frac{1}{\frac{q_t(i)}{q_0(i)}} \cdot \frac{p_t(i)q_t(i)}{\sum_{j=1}^{n} p_t(j)q_t(j)}} = \frac{\sum\limits_{i=1}^{n} q_t(i)p_t(i)}{\sum\limits_{i=1}^{n} q_0(i)p_t(i)}$$

Wertindex: $$I^v_{Pa;0,t} = \frac{1}{\sum_{i=1}^{n} \frac{1}{\frac{v_t(i)}{v_0(i)}} \cdot \frac{v_t(i)}{\sum_{j=1}^{n} v_0(j)}} = \frac{\sum\limits_{i=1}^{n} p_t(i)q_t(i)}{\sum\limits_{i=1}^{n} p_0(i)q_0(i)}$$

5.2.3 Nach Fisher

Preisindex: $I^p_{Fi;0,t} = \sqrt{I^p_{La;0,t} I^p_{Pa;0,t}}$

Mengenindex: $I^q_{Fi;0,t} = \sqrt{I^q_{La;0,t} I^q_{Pa;0,t}}$

Wertindex: $I^v_{Fi;0,t} = \sqrt{I^v_{La;0,t} I^v_{Pa;0,t}}$

5.2.4 Kanonischer Wertindex

$$I^v_{0,t} = \frac{\sum_{i=1}^{n} v_t(i)}{\sum_{i=1} v_0(i)} = \frac{\sum_{i=1}^{n} p_t(i)q_t(i)}{\sum_{i=1}^{n} p_0(i)q_0(i)} = I^v_{La;0,t} = I^v_{Pa;0,t} = I^v_{Fi;0,t}$$

5.2.5 Indexeigenschaften

Probe nach Fisher	Laspeyres	Paasche	Fisher
Identität ($I_{t,t} = 1$)	+	+	+
Zeitumkehr ($I_{t,0} = \frac{1}{I_{0,t}}$)	-	-	+
Rund ($I_{t_1,t_T} = I_{t_1,t_2} I_{t_2,t_3} \dots I_{t_{T-1},t_T}$)	-	-	-
Faktorumkehr ($I^v_{0,t} = I^p_{0,t} I^q_{0,t}$)	-	-	+
Proportionalität[1]	+	+	+
Dimensionswechsel (Unabh. von Preiseinheit)	+	+	+
Bestimmtheit (Def. Preise oder Mengen gleich 0)	+	+	+

[1] Wenn alle $p_t(i) = (1+\alpha)p_0(i) \Rightarrow I^p_{0,t} = 1 + \alpha$

6 Kombinatorik

	ohne Wiederholung	mit Wiederholung
Permutation	$P(n) = n!$	$P(n; g_1, \dots, g_r) = \frac{n!}{g_1! \cdot g_2! \cdot \dots \cdot g_r!}$
Variation	$V(n,k) = \frac{n!}{(n-k)!}$	$V^W(n,k) = n^k$
Kombination	$K(n,k) = \frac{n!}{k! \cdot (n-k)!} = \binom{n}{k}$	$K^W(n,k) = \binom{n+k-1}{k}$

Permutation beliebige Anordnung von n Elementen
Variation Auswahl von k aus n unter Berücksichtigung der Anordnung
Kombination Auswahl von k aus n ohne Berücksichtigung der Anordnung

Binomialkoeffizienten

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!} = \binom{n}{n-k}$$

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

$$\binom{n}{0} = \binom{n}{n} = 1, \qquad \binom{n}{1} = \binom{n}{n-1} = n$$

k\n	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2		1	3	6	10	15	21	28	36	45	55	66	78	91
3			1	4	10	20	35	56	84	120	165	220	286	364
4				1	5	15	35	70	126	210	330	495	715	1001
5					1	6	21	56	126	252	462	792	1287	2002
6						1	7	28	84	210	462	924	1716	3003
7							1	8	36	120	330	792	1716	3432
8								1	9	45	165	495	1287	3003
9									1	10	55	220	715	2002
10										1	11	66	286	1001
11											1	12	78	364
12												1	13	91
13													1	14
14														1

7 Wahrscheinlichkeitsrechnung

7.1 Ereignisse

Beschreibung des zugrundeliegenden Sachverhaltes	**Bezeichnung (Sprechweise)**	**Darstellung**
A tritt sicher ein	A ist sicheres Ereignis	$A = S$
A tritt sicher nicht ein Wenn A eintritt, tritt B ein	A ist unmögliches Ereignis A ist Teilmenge von B	$A = \emptyset$ $A \subset B$
Genau dann, wenn A eintritt, tritt B ein	A und B sind äquivalente Ereignisse	$A \equiv B$
Wenn A eintritt, tritt B nicht ein	A und B sind disjunkte Ereignisse	$A \cap B = \emptyset$
Genau dann, wenn A eintritt, tritt B nicht ein	A und B sind komplementäre Ereignisse	$B = \overline{A}$
Genau dann, wenn mindestens ein A_i eintritt (genau dann, wenn A_1 oder A_2 oder ... eintritt), tritt A ein	A ist Vereinigung der A_i	$A = \bigcup_i A_i$
Genau dann, wenn alle A_i eintreten (genau dann, wenn A_1 und A_2 und ... eintreten), tritt A ein	A ist Durchschnitt der A_i	$A = \bigcap_i A_i$

7.2 Additionssätze

Allgemeine Additionssätze

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ &+ P(A \cap B \cap C) \end{aligned}$$

Additionssatz für disjunkte Ereignisse ($A_i \cap A_j = 0$ für alle $i \neq j$)

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n) = \sum_{i=1}^{n} P(A_i)$$

7.3 Bedingte Wahrscheinlichkeit

$P(A|B) = \frac{P(A \cap B)}{P(B)}$, $\quad P(B) > 0$ und $P(B|A) = \frac{P(A \cap B)}{P(A)}$, $\quad P(A) > 0$

7.4 Unabhängige Ereignisse

$P(A|B) = P(A|\overline{B}) = P(A)$ und $P(B|A) = P(B|\overline{A}) = P(B)$

7.5 Multiplikationssätze

Allgemeiner Multiplikationssatz

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B|A) = P(B) \cdot P(A|B) \\ P(A_1 \cap A_2 \cap A_3) &= P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) \end{aligned}$$

Multipikationssatz für unabhängige Ereignisse

$$P(A \cap B) = P(A) \cdot P(B)$$

7.6 Totale Wahrscheinlichkeit

$$\begin{aligned} P(B) &= P(A_1 \cap B) + P(A_2 \cap B) + \ldots + P(A_n \cap B) = \sum_{i=1}^{n} P(A_i \cap B) \\ &= P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + \ldots + P(B|A_n) \cdot P(A_n) \\ &= \sum_{i=1}^{n} P(B|A_i) \cdot P(A_i) \end{aligned}$$

7.7 Theorem von Bayes

$$P(A_j|B) = \frac{P(B|A_j) \cdot P(A_j)}{\sum_{i=1}^{n} P(B|A_i) \cdot P(A_i)} \quad \forall j = 1, \ldots, n$$

8 Zufallsvariablen

8.1 Verteilung von Zufallsvariablen

8.1.1 Verteilung diskreter Zufallsvariablen

Wahrscheinlichkeitsfunktion einer diskreten Zufallsvariablen

$$f(x_i) = P(X = x_i), \quad (i = 1, 2, \dots)$$

Verteilungsfunktion einer diskreten Zufallsvariablen

$$F(x) = \sum_{x_i \leq x} f(x_i) = P(X \leq x), \quad (i = 1, 2, \dots)$$

8.1.2 Verteilung stetiger Zufallsvariablen

Wahrscheinlichkeitsdichte stetiger Zufallsvariablen

$$\int_a^b f(x)\,dx = P(a < X \leq b), \quad \text{für alle } a, b \text{ mit } a \leq b$$

Verteilungsfunktion stetiger Zufallsvariablen

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = P(-\infty < X \leq x)$$

8.1.3 Berechnung von Wahrscheinlichkeiten

$$\begin{aligned}
P(X \leq a) &= F(a) \\
P(X < a) &= F(a) - P(X = a) \\
P(X > a) &= 1 - F(a) \\
P(X \geq a) &= 1 - F(a) + P(X = a) \\
P(a < X \leq b) &= F(b) - F(a) \\
P(a \leq X \leq b) &= F(b) - F(a) + P(X = a) \\
P(a < X < b) &= F(b) - F(a) - P(X = b) \\
P(a \leq X < b) &= F(b) - F(a) + P(X = a) - P(X = b)
\end{aligned}$$

8.2 Parameter von Zufallsvariablen

8.2.1 Lageparameter

Erwartungswert

diskrete Zufallsvariablen $E[X] = \mu_X = \sum_{i=1}^{k} x_i \cdot f(x_i)$

stetige Zufallsvariablen $E[X] = \mu_X = \int_{-\infty}^{+\infty} x \cdot f(x)dx$

Rechenregeln für Erwartungswerte

$$\begin{aligned}
E[a + b \cdot X] &= a + b \cdot E[X] \ (a, b \text{ konstant}) \\
E[X \pm Y] &= E[X] \pm E[Y]
\end{aligned}$$

8.2.2 Streuungsparameter

Varianz

Diskrete Zufallsvariablen:

$$Var(X) = \sigma_X^2 = \sum_{i=1}^{k} (x_i - \mu_X)^2 \cdot f(x_i) = \sum_{i=1}^{k} x_i^2 \cdot f(x_i) - \mu_X^2$$

Stetige Zufallsvariablen:

$$Var(X) = \sigma_X^2 = \int_{-\infty}^{+\infty} (x - \mu_X)^2 \cdot f(x)\,dx = \int_{-\infty}^{+\infty} x^2 \cdot f(x)\,dx - \mu_X^2$$

Allgemein:

$$Var(X) = \sigma_X^2 = E[(X - E[X])^2] = E\left[X^2\right] - (E[X])^2$$

Rechenregeln für Varianzen

$$\begin{aligned}
Var(a + b \cdot X) &= b^2 \cdot Var(X) \ (a, b \text{ konstant}) \\
Var(X \pm Y) &= Var(X) + Var(Y) \pm 2 \cdot Cov(X, Y)
\end{aligned}$$

8.3 Verteilung von Zufallsvariablen

8.3.1 Zwei diskrete Zufallsvariablen

Gemeinsame Verteilung

Wahrscheinlichkeitsfunktion:

$$P(X = x_i, Y = y_j) = f(x_i, y_j); \quad \text{mit } i = 1, \ldots, m;\ j = 1, \ldots, r$$

Verteilungsfunktion:

$$F(x, y) = P(X \le x, Y \le y) = \sum_{x_i \le x} \sum_{y_j \le y} f(x_i, y_j)$$

Randverteilungen

Wk.funktion für X	$f(x_i) = P(X = x_i) = \sum_{j=1}^{r} f(x_i, y_j)$
Wk.funktion für Y	$f(y_j) = P(Y = y_j) = \sum_{i=1}^{m} f(x_i, y_j)$
Verteilungsfunktion für X	$P(X \le x) = F(x) = \sum_{j=1}^{r} \sum_{x_i \le x} f(x_i, y_j)$
Verteilungsfunktion für Y	$P(Y \le y) = F(y) = \sum_{y_j \le y} \sum_{i=1}^{m} f(x_i, y_j)$

Bedingte Verteilungen

Wk.funktion bedingt auf Y:

$$P(X = x_i | Y = y_j) = \frac{P(X = x_i, Y = y_j)}{P(Y = y_j)} = \frac{f(x_i, y_j)}{f(y_j)} = f(x_i | y_j)$$

Wk.funktion bedingt auf X:

$$P(Y = y_j | X = x_i) = \frac{P(X = x_i, Y = y_j)}{P(X = x_i)} = \frac{f(x_i, y_j)}{f(x_i)} = f(y_j | x_i)$$

8.3.2 Zwei stetige Zufallsvariablen

Gemeinsame Verteilung

Dichtefunktion	$P(x < X \le x + \Delta x; y < Y \le y + \Delta y) = f(x, y)$
Verteilungsfunktion	$F(x, y) = P(X \le x, Y \le y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f(u, v)\, du dv$

Randverteilungen

Dichtefunktion für X	$f(x) = \int_{-\infty}^{+\infty} f(x, y)\, dy$
Dichtefunktion für Y	$f(y) = \int_{-\infty}^{+\infty} f(x, y)\, dx$
Verteilungsfunktion für X	$P(X \le x) = F(x) = \int_{-\infty}^{+\infty} \int_{-\infty}^{x} f(u, v)\, du dv$
Verteilungsfunktion für Y	$P(Y \le y) = F(y) = \int_{-\infty}^{y} \int_{-\infty}^{+\infty} f(u, v)\, du dv$

Bedingte Verteilungen

Wk.funktion bedingt auf Y	$f(x \vert y) = \frac{f(x, y)}{f(y)}$
Wk.funktion bedingt auf X	$f(y \vert x) = \frac{f(x, y)}{f(x)}$

8.4 Unabhängigkeit und Kovarianz für Zufallsvariablen

8.4.1 Unabhängigkeit

Zwei Zufallsvariablen X und Y sind unabhängig, wenn gilt:

diskreter Fall	$f(x_i, y_j) = f(x_i) \cdot f(y_j)$	für alle x_i, y_j
stetiger Fall	$f(x, y) = f(x) \cdot f(y)$	für alle x,y

8.4.2 Kovarianz zweier Zufallsvariablen

$$Cov(X, Y) = E\left[(X - E[X]) \cdot (Y - E[Y])\right] = E[XY] - E[X]\, E[Y]$$

8.4.3 Theoretischer Korrelationskoeffizient

$$\rho(X, Y) = E\left[\frac{(X - E[X])}{\sigma_X} \cdot \frac{(Y - E[Y])}{\sigma_Y}\right] = \frac{Cov(X, Y)}{\sigma_X \cdot \sigma_Y} \quad \text{mit } -1 \le \rho(X, Y) \le +1$$

8.4.4 Linearkombinationen von Zufallsvariablen

Linearkombinationen:

$Z_1 = a \cdot X + b \cdot Y$ $\qquad$ $Z_2 = a \cdot X - b \cdot Y$

Erwartungswerte:

$E[Z_1] = a \cdot E[X] + b \cdot E[Y]$ $\qquad$ $E[Z_2] = a \cdot E[X] - b \cdot E[Y]$

Varianzen:

$$\begin{aligned} Var(Z_1) &= a^2 \cdot Var(X) + b^2 \cdot Var(Y) + 2 \cdot a \cdot b \cdot Cov(X,Y) \\ Var(Z_2) &= a^2 \cdot Var(X) + b^2 \cdot Var(Y) - 2 \cdot a \cdot b \cdot Cov(X,Y) \end{aligned}$$

9 Verteilungsmodelle

9.1 Diskrete Verteilungen

9.1.1 Diskrete Gleichverteilung

$X \sim U(n)$ $\qquad$ $E[X] = \frac{1}{n}\sum_{i=1}^{n} x_i$ $\qquad$ $Var(X) = \frac{1}{n}\sum_{i=1}^{n}(x_i - E[X])^2$

$$f_U(x_i;n) = \begin{cases} \frac{1}{n} & \text{für } i = 1,\ldots,n \\ 0 & \text{sonst} \end{cases}$$

$$F_U(x;n) = \begin{cases} 0 & \text{für } x \le x_1 \\ \frac{i}{n} & \text{für } x_i < x \le x_{i+1} \quad (i = 1,\ldots,n-1) \\ 1 & \text{für } x > x_n \end{cases}$$

9.1.2 Bernoulliverteilung

$X \sim B(p)$ $\qquad$ $E[X] = p$ $\qquad$ $Var(X) = p \cdot (1-p)$

$$f_B(x;p) = \begin{cases} 1-p & \text{für } x = 0 \\ p & \text{für } x = 1 \\ 0 & \text{sonst} \end{cases}$$

$$F_B(x;p) = \begin{cases} 0 & \text{für } x < 0 \\ 1-p & \text{für } 0 \le x < 1 \\ 1 & \text{für } x \ge 1 \end{cases}$$

9.1.3 Binomialverteilung

$X \sim B(n;p)$ $\qquad$ $E[X] = n \cdot p$ $\qquad$ $Var(X) = n \cdot p \cdot (1-p)$

$$f_B(x;n,p) = \begin{cases} \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x} & \text{für } x = 0,1,\ldots,n \\ 0 & \text{sonst} \end{cases}$$

$$F_B(x;n,p) = \begin{cases} \sum_{k=0}^{x} \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} & \text{für } x \ge 0 \\ 0 & \text{für } x < 0 \end{cases}$$

Tabellen für die Verteilungsfunktion $F_B(x;n,p)$ finden sich auf Seite 41ff und es gilt $p > 0{,}5$:
$f_B(x;n;p) = f_B(n-x;n;1-p)$ und $F_B(x;n;p) = 1 - F_B(n-x-1;n;1-p)$

9.1.4 Hypergeometrische Verteilung

$X \sim H(N; M; n)$ $\quad E[X] = n \cdot \frac{M}{N}$ $\quad Var(X) = n \cdot \frac{M}{N} \cdot (1 - \frac{M}{N}) \cdot \left(\frac{N-n}{N-1}\right)$

$$f_H(x; N, M, n) = \begin{cases} \dfrac{\binom{M}{x} \cdot \binom{N-M}{n-x}}{\binom{N}{n}} & \text{für } x \in \{0, 1, \ldots, \min(n, M)\} \\ 0 & \text{sonst} \end{cases}$$

9.1.5 Poisson-Verteilung

$X \sim PO(\lambda)$ $\quad E[X] = \lambda$ $\quad Var(X) = \lambda$

$$f_{PO}(x; \lambda) = \frac{\lambda^x}{x!} \cdot e^{-\lambda} \qquad \text{für } x = 0, 1, 2, \ldots; \lambda > 0$$

$$F_{PO}(x; \lambda) = \begin{cases} \sum_{k=0}^{x} \dfrac{\lambda^k}{k!} \cdot e^{-\lambda} & \text{für } k \geq 0; \lambda > 0 \\ 0 & \text{für } k < 0 \end{cases}$$

Tabellen für die Verteilungsfunktion $F_{PO}(x; \lambda)$ finden sich auf Seite 61ff

9.2 Stetige Verteilungen

9.2.1 Stetige Gleichverteilung

$X \sim U(a; b)$ $\quad E[X] = \frac{b+a}{2}$ $\quad Var(X) = \frac{(b-a)^2}{12}$

$$f_U(x; a; b) = \begin{cases} \dfrac{1}{b-a} & \text{für } a \leq x \leq b \\ 0 & \text{sonst} \end{cases}$$

$$F_U(x; a; b) = \begin{cases} 0 & \text{für } x \leq a \\ \dfrac{x-a}{b-a} & \text{für } a < x \leq b \\ 1 & \text{für } b < x \end{cases}$$

9.2.2 Exponentialverteilung

$X \sim EX(\lambda)$ $\quad E[X] = \frac{1}{\lambda}$ $\quad Var(X) = \frac{1}{\lambda^2}$

$$f_{EX}(x; \lambda) = \begin{cases} \lambda \cdot e^{-\lambda x} & \text{für } x \geq 0, \lambda > 0 \\ 0 & \text{für } x < 0 \end{cases}$$

$$F_{EX}(x; \lambda) = \begin{cases} 1 - e^{-\lambda x} & \text{für } x \geq 0 \\ 0 & \text{für } x < 0 \end{cases}$$

9.2.3 Normalverteilung

$X \sim N(\mu; \sigma)$ $\quad E[X] = \mu$ $\quad Var(X) = \sigma^2$

$$f_N(x; \mu; \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2 \cdot \sigma^2}\right) \qquad \text{für } -\infty < x < +\infty, \sigma > 0$$

$$F_N(x; \mu; \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{(t-\mu)^2}{2 \cdot \sigma^2}\right) dt$$

9.2.4 Standardnormalverteilung

$Z \sim N(0; 1)$ $\quad E[Z] = 0$ $\quad Var(Z) = 1$

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{z^2}{2}\right)$$

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{z} \exp\left(-\frac{v^2}{2}\right) dv$$

Tabelle für die Verteilungsfunktion $\Phi(z)$ ist am Ende der Formelsammlung

Beziehung zwischen Normalverteilung und der Standardnormalverteilung

$$X = \mu + Z \cdot \sigma \quad \text{bzw.} \quad Z = \frac{X - \mu}{\sigma}$$

$$\begin{aligned} P(X \leq x) &= P\left(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right) = P\left(Z \leq \frac{x-\mu}{\sigma}\right) \\ &= \Phi\left(\frac{x-\mu}{\sigma}\right) = \Phi(z) = P(Z \leq z) \end{aligned}$$

9.2.5 Zentraler Grenzwertsatz

$X_1, X_2, \ldots, X_n$ seien unabhängige, identisch verteilte Zufallsvariablen mit $E[X_i] = \mu \neq \pm\infty$ und $Var(X_i) = \sigma^2 < \infty$ (für $i = 1, \ldots, n$). Dann hat die Zufallsvariable $S_n = \Sigma_i X_i$ den Erwartungswert $E[S_n] = n\mu$ und die Varianz $Var(S_n) = n\sigma^2$. Die Verteilung der standardisierten Zufallsvariablen

$$Z_n = \frac{S_n - E[S_n]}{\sqrt{Var(S_n)}} = \frac{\sum_{i=1}^{n} X_i - n \cdot \mu}{\sqrt{n \cdot \sigma^2}} = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{X_i - \mu}{\sigma}$$

konvergiert mit steigendem n gegen die standardisierte Normalverteilung:

$$\lim_{n\to\infty} P(Z_n \leq z) = \Phi(z).$$

9.2.6 χ^2-Verteilung

$X \sim \chi^2_f$ $\quad E[X] = f$ $\quad Var(X) = 2 \cdot f$

Tabellen für die Quantile finden sich auf Seite 67

9.2.7 t-Verteilung

$X \sim t_n$ $\quad E[X] = 0$ für $n > 1$ $\quad Var(X) = \frac{n}{n-2}$ für $n > 2$

Tabellen für die Quantile finden sich auf Seite 69

9.2.8 F-Verteilung

$X \sim F_{f_1;f_2}$ $\quad E[X] = \frac{f_2}{f_2-2}$ für $f_2 > 2$ $\quad Var(X) = \frac{f_2^2(f_1+f_2-2)}{f_1(f_2-2)^2(f_2-4)}$ für $f_2 > 4$

Tabellen für die Quantile finden sich auf Seite 70

9.3 Approximation von Verteilungen

Exakte Verteilung	**Approximations-bedingung(en)**	**Approximative Verteilung**
$X \sim Hyp(N;M;n)$	$\frac{n}{N} < 0,05$	$X \approx B\left(n; p := \frac{M}{N}\right)$
	$\frac{n}{N} < 0,05$, $\frac{M}{N} < 0,05$, $n > 10$	$X \approx Po\left(\lambda := n\frac{M}{N}\right)$
	$n\frac{M}{N}(1 - \frac{M}{N}) \geq 9$	$X \approx N\left(\mu := n\frac{M}{N}; \sigma^2\right)$ $\sigma^2 := n\frac{M}{N}\left(1 - \frac{M}{N}\right)\frac{N-n}{N-1}$
$X \sim B(n;p)$	$p < 0,05$, $n > 10$	$X \approx Po\left(\lambda := np\right)$
	$np(1-p) \geq 9$	$X \approx N\left(\mu := np; \sigma^2\right)$ $\sigma^2 := np(1-p)$
$X \sim Po(\lambda)$	$\lambda \geq 9$	$X \approx N\left(\mu := \lambda; \sigma^2 := \lambda\right)$
$X \sim \chi^2_f$	$f \geq 30$	$X \approx N\left(\mu := f; \sigma^2 := 2f\right)$
$X \sim t_n$	$n \geq 30$	$X \approx N(0;1)$

Die Stetigkeitskorrektur wird bei der Approximation einer diskreten Verteilung durch eine Normalverteilung benutzt, wenn die Varianz σ^2 der Normalverteilung kleiner als 9 ist.

Stetigkeitskorrektur:
X, mit einer diskrete Verteilung, ist approximierbar durch $Y \sim N(\mu, \sigma^2)$ mit $\mu = E(X)$ und $\sigma^2 = Var(X) < 9$. Dann gilt:

$$\begin{aligned}
P(a \leq X \leq b) &\approx P(a - 0,5 \leq Y \leq b + 0,5) \\
P(a < X \leq b) &\approx P(a + 0,5 \leq Y \leq b + 0,5) \\
P(a \leq X < b) &\approx P(a - 0,5 \leq Y \leq b - 0,5) \\
P(a < X < b) &\approx P(a + 0,5 \leq Y \leq b - 0,5) \\
P(X = a) &\approx P(a - 0,5 \leq Y \leq a + 0,5)
\end{aligned}$$

10 Stichprobenverteilung

10.1 Stichprobenverteilung des Stichprobenmittelwertes

Stichprobenvariablen $E[X_i] = \mu$, $Var(X_i) = \sigma^2 \quad (i = 1, \ldots, n)$

Stichprobenfunktion $\overline{X} = \frac{1}{n} \cdot \sum_{i=1}^{n} X_i$, $E[\overline{X}] = \mu$

Stichprobenwert $\overline{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$

Varianz von $\overline{X}$		Varianz der Grundgesamtheit σ^2	
Stichprobe	$\frac{n}{N}$	bekannt	unbekannt
mit Zurücklegen		$\frac{\sigma^2}{n}$	$\frac{S^2}{n}$
ohne Zurücklegen	$< 0,05$	$\frac{\sigma^2}{n}$	$\frac{S^2}{n}$
	$\geq 0,05$	$\frac{\sigma^2}{n} \cdot \frac{(N-n)}{(N-1)}$	$\frac{S^2}{n} \cdot \frac{(N-n)}{(N-1)}$

Verteilung von $\overline{X}$ bei einfacher Zufallsstichprobe

Grundgesamtheit	σ^2	Zufallsvariable	Verteilung	Bedingung
$X_i \sim N(\mu;\sigma)$	bekannt	$Z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$	$N(0,1)$	
	unbekannt	$T = \frac{\overline{X}-\mu}{S/\sqrt{n}}$	$t(n-1)$	für $n \leq 30$
			$\approx N(0,1)$	für $n > 30$
Verteilung unbekannt	bekannt	$Z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$	$\approx N(0,1)$	für $n > 30$
	unbekannt	$T = \frac{\overline{X}-\mu}{S/\sqrt{n}}$	$\approx N(0,1)$	für $n > 30$

10.2 Stichprobenverteilung des Stichprobenanteilswertes

Stichprobenfunktion: $\hat{\Pi} = \frac{X}{n}$ Stichprobenwert: $p = \frac{x}{n}$

Verteilung bei einfacher Zufallsstichprobe

$X \sim B(n;\pi)$ $E[X] = n \cdot \pi$ $Var(X) = n \cdot \pi \cdot (1-\pi)$

Approximation durch die Normalverteilung, wenn die Bedingungen erfüllt sind:

$$\hat{\Pi} \approx N\left(\pi; \sigma_{\hat{\Pi}} = \sqrt{\frac{\pi(1-\pi)}{n}}\right)$$

Verteilung bei uneingeschränkter Zufallsstichprobe

$X \sim H(N;M;n)$ mit $\pi = M/N$ $E[X] = n \cdot \pi$ $Var(X) = n \cdot \pi \cdot (1-\pi) \frac{N-n}{N-1}$

Approximation durch die Normalverteilung, wenn die Bedingungen erfüllt sind:

$$\hat{\Pi} \approx N\left(\pi; \sigma_{\hat{\Pi}} = \sqrt{\frac{\pi(1-\pi)}{n} \frac{N-n}{N-1}}\right)$$

10.3 Stichprobenverteilung der Stichprobenvarianz

Voraussetzung: $X_i \sim N(\mu;\sigma)$ für $i = 1, \ldots, n$

μ	Stichprobenfunktion	Erwartungswert	Verteilung
bekannt	$S^{*2} = \frac{1}{n} \cdot \sum_{i=1}^{n} (X_i - \mu)^2$	$E[S^{*2}] = \sigma^2$	$\frac{n \cdot S^{*2}}{\sigma^2} \sim \chi^2_n$
unbekannt	$S^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (X_i - \overline{X})^2$	$E[S^2] = \sigma^2$	$\frac{(n-1) \cdot S^2}{\sigma^2} \sim \chi^2_{n-1}$

11 Schätzverfahren

11.1 Grundbegriffe

Wahrer Parameter der Grundgesamtheit	ϑ
Schätzfunktion oder Schätzer	$\widehat{\theta} = g(X_1, \dots, X_n)$
Schätzwert	$\hat{\vartheta} = g(x_1, \dots, x_n)$

Mittlere quadratische Abweichung (MSE=Mean Square Error)

$$MSE = E[(\widehat{\theta} - \vartheta)^2] = \underbrace{E[(\widehat{\theta} - E[\widehat{\theta}])^2]}_{=Var(\widehat{\theta})} + \underbrace{(E[\widehat{\theta}] - \vartheta)^2}_{=\text{Verzerrung}^2}$$

Schwankungsintervall ($\widehat{\theta}$ symmetrisch verteilt um ϑ)

$$P(\vartheta - c \cdot \sigma(\widehat{\theta}) \leq \widehat{\theta} \leq \vartheta + c \cdot \sigma(\widehat{\theta})) = 1 - \alpha$$

11.2 Schätzmethoden

11.2.1 Maximum - Likelihood Methode

Likelihood-Funktion	$L(\vartheta) = L(\vartheta \mid x_1, \dots, x_n) = \prod_{i=1}^{n} f(x_i \mid \vartheta)$	$\rightarrow$ maximieren
LogLikelihood-Funktion	$\log(L(\vartheta)) = \sum_{i=1}^{n} \log(f(x_i \mid \vartheta))$	$\rightarrow$ maximieren

11.2.2 Methode der kleinsten Quadrate

Quadratische Form $Q(\vartheta) = \sum_{i=1}^{n}(x_i - E[X_i])^2 = \sum_{i=1}^{n}(x_i - g_i(\vartheta))^2 \rightarrow$ minimieren

11.3 Intervallschätzung

Konfidenzintervall zum Konfidenzniveau $1 - \alpha$

$$P(V_u \leq \vartheta \leq V_o) = P\left(\widehat{\theta} - c \cdot \sigma(\widehat{\theta}) \leq \vartheta \leq \widehat{\theta} + c \cdot \sigma(\widehat{\theta})\right) = 1 - \alpha$$

$$[V_u, V_o] = [\widehat{\theta} - c \cdot \sigma(\widehat{\theta}), \widehat{\theta} + c \cdot \sigma(\widehat{\theta})]$$

11.3.1 Konfidenzintervall für den Erwartungswert μ

Voraussetzung: X_i in der Grundgesamtheit normalverteilt oder Verteilung in Grundgesamtheit unbekannt, aber $n \geq 30$

$Var(X_i) = \sigma^2$ bekannt	
Konfidenzintervall	$P\left(\overline{X} - z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{X} + z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$ $\left[\overline{X} - z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}; \overline{X} + z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right]$
Schätzintervall	$\left[\overline{x} - z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}; \overline{x} + z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right]$
Länge	$\ell = 2 \cdot e = 2 \cdot z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$ mit ℓ = Länge und e = Schätzfehler
Stichprobenumfang	$n \geq \frac{\sigma^2 \cdot z^2_{1-\frac{\alpha}{2}}}{e^2}$

$Var(X_i) = \sigma^2$ unbekannt	
Konfidenzintervall	$P\left(\overline{X} - t_{1-\frac{\alpha}{2};f} \cdot \frac{S}{\sqrt{n}} \leq \mu \leq \overline{X} + t_{1-\frac{\alpha}{2};f} \cdot \frac{S}{\sqrt{n}}\right) = 1 - \alpha$ $\left[\overline{X} - t_{1-\frac{\alpha}{2};f} \cdot \frac{S}{\sqrt{n}}; \overline{X} + t_{1-\frac{\alpha}{2};f} \cdot \frac{S}{\sqrt{n}}\right]$
Schätzintervall	$\left[\overline{x} - t_{1-\frac{\alpha}{2};f} \cdot \frac{s}{\sqrt{n}}; \overline{x} + t_{1-\frac{\alpha}{2};f} \cdot \frac{s}{\sqrt{n}}\right]$
Länge	$\ell = 2 \cdot e = 2 \cdot t_{1-\frac{\alpha}{2};f} \cdot \frac{S}{\sqrt{n}}$
Approximatives Konfidenzintervall für $n > 30$	$P\left(\overline{X} - z_{1-\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}} \leq \mu \leq \overline{X} + z_{1-\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}\right) \approx 1 - \alpha$ $\left[\overline{X} - z_{1-\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}; \overline{X} + z_{1-\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}\right]$

11.3.2 Konfidenzintervall für den Anteilswert π bei Normalapproximation

Voraussetzung	$X \sim B(n;\pi)$ und $\hat{\Pi} = X/n$ ist approximativ normal verteilt
Approximatives Konfidenzintervall	$P\left(\frac{X}{n} - z_{1-\frac{\alpha}{2}} \cdot \sigma_{\hat{\Pi}} \leq \pi \leq \frac{X}{n} + z_{1-\frac{\alpha}{2}} \cdot \sigma_{\hat{\Pi}}\right) = 1-\alpha$ $\left[\frac{X}{n} - z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\frac{X}{n}\cdot\left(1-\frac{X}{n}\right)}{n}}; \frac{X}{n} + z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\frac{X}{n}\cdot\left(1-\frac{X}{n}\right)}{n}}\right]$
Schätzintervall	$\left[\frac{x}{n} - z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\frac{x}{n}\cdot\left(1-\frac{x}{n}\right)}{n}}; \frac{x}{n} + z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\frac{x}{n}\cdot\left(1-\frac{x}{n}\right)}{n}}\right]$
Stichprobenumfang	$n \geq \frac{z_{1-\alpha/2}^2}{4 \cdot e^2}$

12 Testverfahren

12.1 Grundbegriffe

12.1.1 Hypothesen

Test		**Nullhypothese** H_0	**Alternativhypothese** H_1
Allgemein		$\vartheta \in \Theta_0$	$\vartheta \in \Theta_1$
Zweiseitig		$\vartheta = \vartheta_0$	$\vartheta \neq \vartheta_0$
Einseitig	rechtsseitig	$\vartheta \leq \vartheta_0$	$\vartheta > \vartheta_0$
	linksseitig	$\vartheta \geq \vartheta_0$	$\vartheta < \vartheta_0$

12.1.2 Gütefunktion

$$G(\vartheta) = P(\text{“}H_1\text{”}|\vartheta) \text{ mit } \begin{cases} G(\vartheta) \leq \alpha & \text{für alle } \vartheta \in \Theta_0 \\ G(\vartheta) = 1-\beta(\vartheta) & \text{für alle } \vartheta \in \Theta_1 \end{cases}$$

12.2 Einstichprobentest für μ

Varianz σ^2 der Grundgesamtheit		bekannt	unbekannt
Teststatistik V		$\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}}$	$\frac{\overline{X}-\mu_0}{S/\sqrt{n}}$
Bedingungen		Verteilung von V unter H_0	
$X_i \sim N(\mu;\sigma)$	$n \leq 30$	$N(0,1)$	t_{n-1}
	$n > 30$	$N(0,1)$	
beliebig verteilt	$n > 30$	$\approx N(0,1)$	

12.2.1 Gütefunktion beim Test auf μ

$G(\mu)$ für zweiseitigen Test	
$1 - \left[P\left(V \leq z_{1-\frac{\alpha}{2}} - \frac{\mu-\mu_0}{\sigma/\sqrt{n}}\right) - P\left(V < -z_{1-\frac{\alpha}{2}} - \frac{\mu-\mu_0}{\sigma/\sqrt{n}}\right)\right]$	
$G(\mu)$ für linksseitigen Test	$G(\mu)$ für rechtsseitigen Test
$P\left(V < -z_{1-\alpha} - \frac{\mu-\mu_0}{\sigma/\sqrt{n}}\right)$	$1 - P\left(V \leq z_{1-\alpha} - \frac{\mu-\mu_0}{\sigma/\sqrt{n}}\right)$

12.3 Einstichprobentest für π bei Normalapproximation

$$V = \frac{\widehat{\pi} - \pi_0}{\sqrt{\frac{\pi_0 \cdot (1-\pi_0)}{n}}} \qquad \text{ist unter } H_0 \; N(0,1) \text{ verteilt}$$

12.4 Test für die Differenz zweier Erwartungswerte

Voraussetzung: $X_{1i} \sim N(\mu_1, \sigma_1)$, $X_{2i} \sim N(\mu_2, \sigma_2)$ und $\omega_0 := \mu_1 - \mu_2$

σ_1, σ_2		Teststatistik V
bekannt		$\frac{D-\omega_0}{\sigma_D} = \frac{(\bar{X}_1 - \bar{X}_2) - \omega_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ Verteilung unter H_0: $V \sim N(0,1)$
unbekannt	$\sigma_1^2 = \sigma_2^2$	$\frac{D-\omega_0}{S_D} = \frac{(\bar{X}_1 - \bar{X}_2) - \omega_0}{\sqrt{\frac{n_1+n_2}{n_1 \cdot n_2} \cdot \frac{(n_1-1)\cdot S_1^2 + (n_2-1)\cdot S_2^2}{n_1+n_2-2}}}$ Verteilung unter H_0: $V \sim t_{n_1+n_2-2}$
	$\sigma_1^2 \neq \sigma_2^2$	$\frac{D-\omega_0}{S_D} = \frac{(\bar{X}_1 - \bar{X}_2) - \omega_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$ Verteilung unter H_0: $V \approx t_f \quad f = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\cdot\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\cdot\left(\frac{s_2^2}{n_2}\right)^2}$

Hinweis: Für $n_1 > 30$ und $n_2 > 30$ gilt: $V \approx N(0;1)$

12.5 χ^2 - Anpassungstest

Voraussetzungen	$n \cdot p_i \geq 1$ für alle i und $n \cdot p_i \geq 5$ für mindestens 80% der $n \cdot p_i$
Teststatistik	$V = \sum_{i=1}^{I} \frac{(h_i - n \cdot p_i)^2}{n \cdot p_i} \approx \chi^2_{I-1-k}$ mit k = Zahl der Parameter, die geschätzt werden müssen

12.6 χ^2 - Unabhängigkeitstest

Voraussetzungen	$\hat{h}_{ij} \geq 5$ für alle i und j
Teststatistik	$V = \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{(h_{ij} - \hat{h}_{ij})^2}{\hat{h}_{ij}} \approx \chi^2_f \quad$ mit $f = (I-1)(J-1)$, wobei I = Anzahl Zeilen, J = Anzahl Spalten, $\hat{h}_{ij} = h_{i\bullet} h_{\bullet j}/n$

13 Regressionsanalyse

13.1 Allgemeines Regressionsmodell

$$Y_i = f(x_{1i}, x_{2i}, ..., x_{mi}) + U_i = E[Y_i] + U_i \qquad \text{mit } E[U_i] = 0$$

13.2 Einfache lineare Regressionfunktion

Wahre Regressionsgerade	$E[Y_i] = \beta_0 + \beta_1 \cdot x_i$
Regressionsmodell	$Y_i = E[Y_i] + U_i = \beta_0 + \beta_1 \cdot x_i + U_i$
Störterm	$U_i = Y_i - E[Y_i]$ mit $E[U_i] = 0$, $Var(U_i) = \sigma_u^2$, $Cov(U_i U_j) = 0$ für $i \neq j$ und $U_i \sim N(0; \sigma_u^2)$
Geschätzte Regressionsgerade	$\hat{y}_i = b_0 + b_1 \cdot x_i$
Stichprobenregressionsmodell	$y_i = \hat{y}_i + \hat{u}_i = b_0 + b_1 \cdot x_i + \hat{u}_i$
Residuen	$\hat{u}_i = y_i - \hat{y}_i$

13.2.1 Kleinste-Quadrate Schätzwerte für $\beta_0, \beta_1, \sigma_u^2$

$$b_1 = \frac{\sum_{i=1}^{n}(x_i - \overline{x}) \cdot (y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2} = \frac{n \cdot \sum_{i=1}^{n} x_i \cdot y_i - \left(\sum_{i=1}^{n} x_i\right) \cdot \left(\sum_{i=1}^{n} y_i\right)}{n \cdot \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$= \frac{s_{xy}}{s_x^2} = r_{xy} \cdot \frac{s_y}{s_x}$$

$$b_0 = \frac{\sum_{i=1}^{n} y_i \cdot \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} x_i \cdot y_i}{n \cdot \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} = \overline{y} - b_1 \cdot \overline{x}$$

$$s_{\hat{u}}^2 = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{n-2}$$

13.2.2 Eigenschaften der KQ-Schätzer

Erwartungswerte:

$$E[b_1] = \beta_1 \qquad E[b_0] = \beta_0$$

Varianzen:

$$Var(b_1) = \sigma_{b_1}^2 = \frac{\sigma_u^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2} \qquad Var(b_0) = \sigma_{b_0}^2 = \frac{\sigma_u^2 \cdot \sum_{i=1}^{n} x_i^2}{n \cdot \sum_{i=1}^{n}(x_i - \overline{x})^2}$$

Geschätzte Varianzen:

$$\hat{\sigma}_{b_1}^2 = \frac{s_u^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2} \qquad \hat{\sigma}_{b_0}^2 = \frac{s_u^2 \cdot \sum_{i=1}^{n} x_i^2}{n \cdot \sum_{i=1}^{n}(x_i - \overline{x})^2}$$

13.2.3 Stichprobenverteilung der KQ-Schätzer falls $U_i \sim N(0, \sigma_u^2)$

$$b_0 \sim N(\beta_0, \sigma_{b_0}^2) \qquad \frac{b_0 - \beta_0}{\hat{\sigma}_{b_0}} \sim t_{n-2}$$

$$b_1 \sim N(\beta_1, \sigma_{b_1}^2) \qquad \frac{b_1 - \beta_1}{\hat{\sigma}_{b_1}} \sim t_{n-2}$$

13.2.4 Test für β_1

Hypothesen	$H_0 : \beta_1 = 0$ vs. $H_1 : \beta_1 \neq 0$
Teststatistik	$V = \frac{b_1}{\hat{\sigma}_{b_1}}$ und verwerfe H_0 falls $\lvert v \rvert > t_{1-\frac{\alpha}{2};n-2}$

13.2.5 Konfidenzintervalle

Für β_0	$[b_0 - t_{1-\frac{\alpha}{2};n-2} \cdot \hat{\sigma}_{b_0}; b_0 + t_{1-\frac{\alpha}{2};n-2} \cdot \hat{\sigma}_{b_0}]$
Für β_1	$[b_1 - t_{1-\frac{\alpha}{2};n-2} \cdot \hat{\sigma}_{b_1}; b_1 + t_{1-\frac{\alpha}{2};n-2} \cdot \hat{\sigma}_{b_1}]$
Für $E[Y]$ an der Stelle x_0	$\left[b_0 + b_1 x_0 \pm t_{1-\frac{\alpha}{2};n-2} \cdot \hat{\sigma}_u \sqrt{\frac{1}{n} + \frac{(x_0 - \overline{x})^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2}}\right]$

14 Verteilungstabellen

14.1 Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,05$

$x\backslash n$	1	2	3	4	5	6	7	8
0	0.9500	0.9025	0.8574	0.8145	0.7738	0.7351	0.6983	0.6634
1	1.0000	0.9975	0.9928	0.9860	0.9774	0.9672	0.9556	0.9428
2	1.0000	1.0000	0.9999	0.9995	0.9988	0.9978	0.9962	0.9942
3	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996
4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	17	18	19	20	21	22	23	24
0	0.4181	0.3972	0.3774	0.3585	0.3406	0.3235	0.3074	0.2920
1	0.7922	0.7735	0.7547	0.7358	0.7170	0.6982	0.6794	0.6608
2	0.9497	0.9419	0.9335	0.9245	0.9151	0.9052	0.8948	0.8841
3	0.9912	0.9891	0.9868	0.9841	0.9811	0.9778	0.9742	0.9702
4	0.9988	0.9985	0.9980	0.9974	0.9968	0.9960	0.9951	0.9940
5	0.9999	0.9998	0.9998	0.9997	0.9996	0.9994	0.9992	0.9990
6	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	33	34	35	36	37	38	39	40
0	0.1840	0.1748	0.1661	0.1578	0.1499	0.1424	0.1353	0.1285
1	0.5036	0.4877	0.4720	0.4567	0.4418	0.4272	0.4129	0.3991
2	0.7728	0.7593	0.7458	0.7321	0.7183	0.7045	0.6906	0.6767
3	0.9192	0.9119	0.9042	0.8963	0.8881	0.8796	0.8709	0.8619
4	0.9770	0.9741	0.9710	0.9676	0.9641	0.9603	0.9562	0.9520
5	0.9946	0.9937	0.9927	0.9917	0.9905	0.9891	0.9877	0.9861
6	0.9989	0.9987	0.9985	0.9982	0.9979	0.9975	0.9971	0.9966
7	0.9998	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994	0.9993
8	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,05$

$x\backslash n$	9	10	11	12	13	14	15	16
0	0.6302	0.5987	0.5688	0.5404	0.5133	0.4877	0.4633	0.4401
1	0.9288	0.9139	0.8981	0.8816	0.8646	0.8470	0.8290	0.8108
2	0.9916	0.9885	0.9848	0.9804	0.9755	0.9699	0.9638	0.9571
3	0.9994	0.9990	0.9984	0.9978	0.9969	0.9958	0.9945	0.9930
4	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9991
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	25	26	27	28	29	30	31	32
0	0.2774	0.2635	0.2503	0.2378	0.2259	0.2146	0.2039	0.1937
1	0.6424	0.6241	0.6061	0.5883	0.5708	0.5535	0.5366	0.5200
2	0.8729	0.8614	0.8495	0.8373	0.8249	0.8122	0.7992	0.7861
3	0.9659	0.9613	0.9563	0.9509	0.9452	0.9392	0.9329	0.9262
4	0.9928	0.9915	0.9900	0.9883	0.9864	0.9844	0.9821	0.9796
5	0.9988	0.9985	0.9981	0.9977	0.9973	0.9967	0.9961	0.9954
6	0.9998	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	0.9991
7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	41	42	43	44	45	46	47	48
0	0.1221	0.1160	0.1102	0.1047	0.0994	0.0945	0.0897	0.0853
1	0.3855	0.3724	0.3595	0.3471	0.3350	0.3232	0.3117	0.3006
2	0.6629	0.6490	0.6352	0.6214	0.6077	0.5940	0.5805	0.5670
3	0.8526	0.8431	0.8334	0.8235	0.8134	0.8031	0.7926	0.7820
4	0.9475	0.9427	0.9377	0.9325	0.9271	0.9214	0.9155	0.9093
5	0.9844	0.9826	0.9806	0.9784	0.9761	0.9737	0.9711	0.9683
6	0.9961	0.9955	0.9949	0.9941	0.9934	0.9925	0.9916	0.9905
7	0.9992	0.9990	0.9988	0.9986	0.9984	0.9982	0.9979	0.9976
8	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994
9	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,10$

$x \backslash n$	1	2	3	4	5	6	7	8
0	0.9000	0.8100	0.7290	0.6561	0.5905	0.5314	0.4783	0.4305
1	1.0000	0.9900	0.9720	0.9477	0.9185	0.8857	0.8503	0.8131
2	1.0000	1.0000	0.9990	0.9963	0.9914	0.9842	0.9743	0.9619
3	1.0000	1.0000	1.0000	0.9999	0.9995	0.9987	0.9973	0.9950
4	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x \backslash n$	17	18	19	20	21	22	23	24
0	0.1668	0.1501	0.1351	0.1216	0.1094	0.0985	0.0886	0.0798
1	0.4818	0.4503	0.4203	0.3917	0.3647	0.3392	0.3151	0.2925
2	0.7618	0.7338	0.7054	0.6769	0.6484	0.6200	0.5920	0.5643
3	0.9174	0.9018	0.8850	0.8670	0.8480	0.8281	0.8073	0.7857
4	0.9779	0.9718	0.9648	0.9568	0.9478	0.9379	0.9269	0.9149
5	0.9953	0.9936	0.9914	0.9887	0.9856	0.9818	0.9774	0.9723
6	0.9992	0.9988	0.9983	0.9976	0.9967	0.9956	0.9942	0.9925
7	0.9999	0.9998	0.9997	0.9996	0.9994	0.9991	0.9988	0.9983
8	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,10$

$x \backslash n$	9	10	11	12	13	14	15	16
0	0.3874	0.3487	0.3138	0.2824	0.2542	0.2288	0.2059	0.1853
1	0.7748	0.7361	0.6974	0.6590	0.6213	0.5846	0.5490	0.5147
2	0.9470	0.9298	0.9104	0.8891	0.8661	0.8416	0.8159	0.7892
3	0.9917	0.9872	0.9815	0.9744	0.9658	0.9559	0.9444	0.9316
4	0.9991	0.9984	0.9972	0.9957	0.9935	0.9908	0.9873	0.9830
5	0.9999	0.9999	0.9997	0.9995	0.9991	0.9985	0.9978	0.9967
6	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x \backslash n$	25	26	27	28	29	30	31	32
0	0.0718	0.0646	0.0581	0.0523	0.0471	0.0424	0.0382	0.0343
1	0.2712	0.2513	0.2326	0.2152	0.1989	0.1837	0.1696	0.1564
2	0.5371	0.5105	0.4846	0.4594	0.4350	0.4114	0.3886	0.3667
3	0.7636	0.7409	0.7179	0.6946	0.6710	0.6474	0.6238	0.6003
4	0.9020	0.8882	0.8734	0.8579	0.8416	0.8245	0.8068	0.7885
5	0.9666	0.9601	0.9529	0.9450	0.9363	0.9268	0.9166	0.9056
6	0.9905	0.9881	0.9853	0.9821	0.9784	0.9742	0.9694	0.9642
7	0.9977	0.9970	0.9961	0.9950	0.9938	0.9922	0.9904	0.9883
8	0.9995	0.9994	0.9991	0.9988	0.9984	0.9980	0.9974	0.9967
9	0.9999	0.9999	0.9998	0.9998	0.9997	0.9995	0.9994	0.9992
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Beispiel: Die Zufallsvariable $X \sim B(13; 0,1)$ und gesucht ist

$$\begin{aligned} P(X = 3) &= F(3) - F(2) = 0,9658 - 0,8661 = 0,0997 \\ P(1 \leq X \leq 3) &= F(3) - F(0) = 0,9658 - 0,2545 = 0,7113 \\ P(X > 2) &= 1 - F(2) = 1 - 0,8661 = 0,1339 \end{aligned}$$

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,15$

$x \backslash n$	1	2	3	4	5	6	7	8
0	0.8500	0.7225	0.6141	0.5220	0.4437	0.3771	0.3206	0.2725
1	1.0000	0.9775	0.9392	0.8905	0.8352	0.7765	0.7166	0.6572
2	1.0000	1.0000	0.9966	0.9880	0.9734	0.9527	0.9262	0.8948
3	1.0000	1.0000	1.0000	0.9995	0.9978	0.9941	0.9879	0.9786
4	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9988	0.9971
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x \backslash n$	17	18	19	20	21	22	23	24
0	0.0631	0.0536	0.0456	0.0388	0.0329	0.0280	0.0238	0.0202
1	0.2525	0.2241	0.1985	0.1756	0.1550	0.1367	0.1204	0.1059
2	0.5198	0.4797	0.4413	0.4049	0.3705	0.3382	0.3080	0.2798
3	0.7556	0.7202	0.6841	0.6477	0.6113	0.5752	0.5396	0.5049
4	0.9013	0.8794	0.8556	0.8298	0.8025	0.7738	0.7440	0.7134
5	0.9681	0.9581	0.9463	0.9327	0.9173	0.9001	0.8811	0.8606
6	0.9917	0.9882	0.9837	0.9781	0.9713	0.9632	0.9537	0.9428
7	0.9983	0.9973	0.9959	0.9941	0.9917	0.9886	0.9848	0.9801
8	0.9997	0.9995	0.9992	0.9987	0.9980	0.9970	0.9958	0.9941
9	1.0000	0.9999	0.9999	0.9998	0.9996	0.9993	0.9990	0.9985
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,15$

$x \backslash n$	9	10	11	12	13	14	15	16
0	0.2316	0.1969	0.1673	0.1422	0.1209	0.1028	0.0874	0.0743
1	0.5995	0.5443	0.4922	0.4435	0.3983	0.3567	0.3186	0.2839
2	0.8591	0.8202	0.7788	0.7358	0.6920	0.6479	0.6042	0.5614
3	0.9661	0.9500	0.9306	0.9078	0.8820	0.8535	0.8227	0.7899
4	0.9944	0.9901	0.9841	0.9761	0.9658	0.9533	0.9383	0.9209
5	0.9994	0.9986	0.9973	0.9954	0.9925	0.9885	0.9832	0.9765
6	1.0000	0.9999	0.9997	0.9993	0.9987	0.9978	0.9964	0.9944
7	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9989
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x \backslash n$	25	26	27	28	29	30	31	32
0	0.0172	0.0146	0.0124	0.0106	0.0090	0.0076	0.0065	0.0055
1	0.0931	0.0817	0.0716	0.0627	0.0549	0.0480	0.0420	0.0366
2	0.2537	0.2296	0.2074	0.1871	0.1684	0.1514	0.1359	0.1218
3	0.4711	0.4385	0.4072	0.3772	0.3487	0.3217	0.2961	0.2721
4	0.6821	0.6505	0.6187	0.5869	0.5555	0.5245	0.4940	0.4644
5	0.8385	0.8150	0.7903	0.7646	0.7379	0.7106	0.6827	0.6544
6	0.9305	0.9167	0.9014	0.8848	0.8667	0.8474	0.8269	0.8053
7	0.9745	0.9679	0.9602	0.9514	0.9414	0.9302	0.9178	0.9042
8	0.9920	0.9894	0.9862	0.9823	0.9777	0.9722	0.9659	0.9587
9	0.9979	0.9970	0.9958	0.9944	0.9926	0.9903	0.9876	0.9844
10	0.9995	0.9993	0.9989	0.9985	0.9978	0.9971	0.9961	0.9948
11	0.9999	0.9998	0.9998	0.9996	0.9995	0.9992	0.9989	0.9985
12	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,20$

$x\backslash n$	1	2	3	4	5	6	7	8
0	0.8000	0.6400	0.5120	0.4096	0.3277	0.2621	0.2097	0.1678
1	1.0000	0.9600	0.8960	0.8192	0.7373	0.6554	0.5767	0.5033
2	1.0000	1.0000	0.9920	0.9728	0.9421	0.9011	0.8520	0.7969
3	1.0000	1.0000	1.0000	0.9984	0.9933	0.9830	0.9667	0.9437
4	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9953	0.9896
5	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9988
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	17	18	19	20	21	22	23	24
0	0.0225	0.0180	0.0144	0.0115	0.0092	0.0074	0.0059	0.0047
1	0.1182	0.0991	0.0829	0.0692	0.0576	0.0480	0.0398	0.0331
2	0.3096	0.2713	0.2369	0.2061	0.1787	0.1545	0.1332	0.1145
3	0.5489	0.5010	0.4551	0.4114	0.3704	0.3320	0.2965	0.2639
4	0.7582	0.7164	0.6733	0.6296	0.5860	0.5429	0.5007	0.4599
5	0.8943	0.8671	0.8369	0.8042	0.7693	0.7326	0.6947	0.6559
6	0.9623	0.9487	0.9324	0.9133	0.8915	0.8670	0.8402	0.8111
7	0.9891	0.9837	0.9767	0.9679	0.9569	0.9439	0.9285	0.9108
8	0.9974	0.9957	0.9933	0.9900	0.9856	0.9799	0.9727	0.9638
9	0.9995	0.9991	0.9984	0.9974	0.9959	0.9939	0.9911	0.9874
10	0.9999	0.9998	0.9997	0.9994	0.9990	0.9984	0.9975	0.9962
11	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9990
12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,20$

$x\backslash n$	9	10	11	12	13	14	15	16
0	0.1342	0.1074	0.0859	0.0687	0.0550	0.0440	0.0352	0.0281
1	0.4362	0.3758	0.3221	0.2749	0.2336	0.1979	0.1671	0.1407
2	0.7382	0.6778	0.6174	0.5583	0.5017	0.4481	0.3980	0.3518
3	0.9144	0.8791	0.8389	0.7946	0.7473	0.6982	0.6482	0.5981
4	0.9804	0.9672	0.9496	0.9274	0.9009	0.8702	0.8358	0.7982
5	0.9969	0.9936	0.9883	0.9806	0.9700	0.9561	0.9389	0.9183
6	0.9997	0.9991	0.9980	0.9961	0.9930	0.9884	0.9819	0.9733
7	1.0000	0.9999	0.9998	0.9994	0.9988	0.9976	0.9958	0.9930
8	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9985
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	25	26	27	28	29	30	31	32
0	0.0038	0.0030	0.0024	0.0019	0.0015	0.0012	0.0010	0.0008
1	0.0274	0.0227	0.0187	0.0155	0.0128	0.0105	0.0087	0.0071
2	0.0982	0.0841	0.0718	0.0612	0.0520	0.0442	0.0374	0.0317
3	0.2340	0.2068	0.1823	0.1602	0.1404	0.1227	0.1070	0.0931
4	0.4207	0.3833	0.3480	0.3149	0.2839	0.2552	0.2287	0.2044
5	0.6167	0.5775	0.5387	0.5005	0.4634	0.4275	0.3931	0.3602
6	0.7800	0.7474	0.7134	0.6784	0.6429	0.6070	0.5711	0.5355
7	0.8909	0.8687	0.8444	0.8182	0.7903	0.7608	0.7300	0.6982
8	0.9532	0.9408	0.9263	0.9100	0.8916	0.8713	0.8492	0.8254
9	0.9827	0.9768	0.9696	0.9609	0.9507	0.9389	0.9254	0.9102
10	0.9944	0.9921	0.9890	0.9851	0.9803	0.9744	0.9673	0.9589
11	0.9985	0.9977	0.9965	0.9950	0.9931	0.9905	0.9873	0.9833
12	0.9996	0.9994	0.9990	0.9985	0.9978	0.9969	0.9956	0.9939
13	0.9999	0.9999	0.9998	0.9996	0.9994	0.9991	0.9987	0.9980
14	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994
15	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,25$

$x\backslash n$	1	2	3	4	5	6	7	8
0	0.7500	0.5625	0.4219	0.3164	0.2373	0.1780	0.1335	0.1001
1	1.0000	0.9375	0.8438	0.7383	0.6328	0.5339	0.4449	0.3671
2	1.0000	1.0000	0.9844	0.9492	0.8965	0.8306	0.7564	0.6785
3	1.0000	1.0000	1.0000	0.9961	0.9844	0.9624	0.9294	0.8862
4	1.0000	1.0000	1.0000	1.0000	0.9990	0.9954	0.9871	0.9727
5	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9958
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	17	18	19	20	21	22	23	24
0	0.0075	0.0056	0.0042	0.0032	0.0024	0.0018	0.0013	0.0010
1	0.0501	0.0395	0.0310	0.0243	0.0190	0.0149	0.0116	0.0090
2	0.1637	0.1353	0.1113	0.0913	0.0745	0.0606	0.0492	0.0398
3	0.3530	0.3057	0.2631	0.2252	0.1917	0.1624	0.1370	0.1150
4	0.5739	0.5187	0.4654	0.4148	0.3674	0.3235	0.2832	0.2466
5	0.7653	0.7175	0.6678	0.6172	0.5666	0.5168	0.4685	0.4222
6	0.8929	0.8610	0.8251	0.7858	0.7436	0.6994	0.6537	0.6074
7	0.9598	0.9431	0.9225	0.8982	0.8701	0.8385	0.8037	0.7662
8	0.9876	0.9807	0.9713	0.9591	0.9439	0.9254	0.9037	0.8787
9	0.9969	0.9946	0.9911	0.9861	0.9794	0.9705	0.9592	0.9453
10	0.9994	0.9988	0.9977	0.9961	0.9936	0.9900	0.9851	0.9787
11	0.9999	0.9998	0.9995	0.9991	0.9983	0.9971	0.9954	0.9928
12	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9988	0.9979
13	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9995
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,25$

$x\backslash n$	9	10	11	12	13	14	15	16
0	0.0751	0.0563	0.0422	0.0317	0.0238	0.0178	0.0134	0.0100
1	0.3003	0.2440	0.1971	0.1584	0.1267	0.1010	0.0802	0.0635
2	0.6007	0.5256	0.4552	0.3907	0.3326	0.2811	0.2361	0.1971
3	0.8343	0.7759	0.7133	0.6488	0.5843	0.5213	0.4613	0.4050
4	0.9511	0.9219	0.8854	0.8424	0.7940	0.7415	0.6865	0.6302
5	0.9900	0.9803	0.9657	0.9456	0.9198	0.8883	0.8516	0.8103
6	0.9987	0.9965	0.9924	0.9857	0.9757	0.9617	0.9434	0.9204
7	0.9999	0.9996	0.9988	0.9972	0.9944	0.9897	0.9827	0.9729
8	1.0000	1.0000	0.9999	0.9996	0.9990	0.9978	0.9958	0.9925
9	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9984
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	25	26	27	28	29	30	31	32
0	0.0008	0.0006	0.0004	0.0003	0.0002	0.0002	0.0001	0.0001
1	0.0070	0.0055	0.0042	0.0033	0.0025	0.0020	0.0015	0.0012
2	0.0321	0.0258	0.0207	0.0166	0.0133	0.0106	0.0084	0.0067
3	0.0962	0.0802	0.0666	0.0551	0.0455	0.0374	0.0307	0.0252
4	0.2137	0.1844	0.1583	0.1354	0.1153	0.0979	0.0828	0.0698
5	0.3783	0.3371	0.2989	0.2638	0.2317	0.2026	0.1764	0.1530
6	0.5611	0.5154	0.4708	0.4279	0.3868	0.3481	0.3117	0.2779
7	0.7265	0.6852	0.6427	0.5997	0.5568	0.5143	0.4727	0.4325
8	0.8506	0.8195	0.7859	0.7501	0.7125	0.6736	0.6338	0.5935
9	0.9287	0.9091	0.8867	0.8615	0.8337	0.8034	0.7710	0.7367
10	0.9703	0.9599	0.9472	0.9321	0.9145	0.8943	0.8716	0.8464
11	0.9893	0.9845	0.9784	0.9706	0.9610	0.9493	0.9356	0.9196
12	0.9966	0.9948	0.9922	0.9888	0.9842	0.9784	0.9711	0.9622
13	0.9991	0.9985	0.9976	0.9962	0.9944	0.9918	0.9885	0.9841
14	0.9998	0.9996	0.9993	0.9989	0.9982	0.9973	0.9959	0.9940
15	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9980
16	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994
17	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,30$

$x\backslash n$	1	2	3	4	5	6	7	8
0	0.7000	0.4900	0.3430	0.2401	0.1681	0.1176	0.0824	0.0576
1	1.0000	0.9100	0.7840	0.6517	0.5282	0.4202	0.3294	0.2553
2	1.0000	1.0000	0.9730	0.9163	0.8369	0.7443	0.6471	0.5518
3	1.0000	1.0000	1.0000	0.9919	0.9692	0.9295	0.8740	0.8059
4	1.0000	1.0000	1.0000	1.0000	0.9976	0.9891	0.9712	0.9420
5	1.0000	1.0000	1.0000	1.0000	1.0000	0.9993	0.9962	0.9887
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	17	18	19	20	21	22	23	24
0	0.0023	0.0016	0.0011	0.0008	0.0006	0.0004	0.0003	0.0002
1	0.0193	0.0142	0.0104	0.0076	0.0056	0.0041	0.0030	0.0022
2	0.0774	0.0600	0.0462	0.0355	0.0271	0.0207	0.0157	0.0119
3	0.2019	0.1646	0.1332	0.1071	0.0856	0.0681	0.0538	0.0424
4	0.3887	0.3327	0.2822	0.2375	0.1984	0.1645	0.1356	0.1111
5	0.5968	0.5344	0.4739	0.4164	0.3627	0.3134	0.2688	0.2288
6	0.7752	0.7217	0.6655	0.6080	0.5505	0.4942	0.4399	0.3886
7	0.8954	0.8593	0.8180	0.7723	0.7230	0.6713	0.6181	0.5647
8	0.9597	0.9404	0.9161	0.8867	0.8523	0.8135	0.7709	0.7250
9	0.9873	0.9790	0.9674	0.9520	0.9324	0.9084	0.8799	0.8472
10	0.9968	0.9939	0.9895	0.9829	0.9736	0.9613	0.9454	0.9258
11	0.9993	0.9986	0.9972	0.9949	0.9913	0.9860	0.9786	0.9686
12	0.9999	0.9997	0.9994	0.9987	0.9976	0.9957	0.9928	0.9885
13	1.0000	1.0000	0.9999	0.9997	0.9994	0.9989	0.9979	0.9964
14	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9990
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,30$

$x\backslash n$	9	10	11	12	13	14	15	16
0	0.0404	0.0282	0.0198	0.0138	0.0097	0.0068	0.0047	0.0033
1	0.1960	0.1493	0.1130	0.0850	0.0637	0.0475	0.0353	0.0261
2	0.4628	0.3828	0.3127	0.2528	0.2025	0.1608	0.1268	0.0994
3	0.7297	0.6496	0.5696	0.4925	0.4206	0.3552	0.2969	0.2459
4	0.9012	0.8497	0.7897	0.7237	0.6543	0.5842	0.5155	0.4499
5	0.9747	0.9527	0.9218	0.8822	0.8346	0.7805	0.7216	0.6598
6	0.9957	0.9894	0.9784	0.9614	0.9376	0.9067	0.8689	0.8247
7	0.9996	0.9984	0.9957	0.9905	0.9818	0.9685	0.9500	0.9256
8	1.0000	0.9999	0.9994	0.9983	0.9960	0.9917	0.9848	0.9743
9	1.0000	1.0000	1.0000	0.9998	0.9993	0.9983	0.9963	0.9929
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9984
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	25	26	27	28	29	30	31	32
0	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0016	0.0011	0.0008	0.0006	0.0004	0.0003	0.0002	0.0002
2	0.0090	0.0067	0.0051	0.0038	0.0028	0.0021	0.0016	0.0012
3	0.0332	0.0260	0.0202	0.0157	0.0121	0.0093	0.0072	0.0055
4	0.0905	0.0733	0.0591	0.0474	0.0379	0.0302	0.0239	0.0189
5	0.1935	0.1626	0.1358	0.1128	0.0932	0.0766	0.0627	0.0510
6	0.3407	0.2965	0.2563	0.2202	0.1880	0.1595	0.1346	0.1131
7	0.5118	0.4605	0.4113	0.3648	0.3214	0.2814	0.2448	0.2118
8	0.6769	0.6274	0.5773	0.5275	0.4787	0.4315	0.3865	0.3440
9	0.8106	0.7705	0.7276	0.6825	0.6360	0.5888	0.5416	0.4951
10	0.9022	0.8747	0.8434	0.8087	0.7708	0.7304	0.6879	0.6440
11	0.9558	0.9397	0.9202	0.8972	0.8706	0.8407	0.8076	0.7717
12	0.9825	0.9745	0.9641	0.9509	0.9348	0.9155	0.8931	0.8674
13	0.9940	0.9906	0.9857	0.9792	0.9707	0.9599	0.9466	0.9306
14	0.9982	0.9970	0.9950	0.9923	0.9883	0.9831	0.9761	0.9673
15	0.9995	0.9991	0.9985	0.9975	0.9959	0.9936	0.9905	0.9862
16	0.9999	0.9998	0.9996	0.9993	0.9987	0.9979	0.9966	0.9948
17	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9989	0.9982
18	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,35$

$x\backslash n$	1	2	3	4	5	6	7	8
0	0.6500	0.4225	0.2746	0.1785	0.1160	0.0754	0.0490	0.0319
1	1.0000	0.8775	0.7183	0.5630	0.4284	0.3191	0.2338	0.1691
2	1.0000	1.0000	0.9571	0.8735	0.7648	0.6471	0.5323	0.4278
3	1.0000	1.0000	1.0000	0.9850	0.9460	0.8826	0.8002	0.7064
4	1.0000	1.0000	1.0000	1.0000	0.9947	0.9777	0.9444	0.8939
5	1.0000	1.0000	1.0000	1.0000	1.0000	0.9982	0.9910	0.9747
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9994	0.9964
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	17	18	19	20	21	22	23	24
0	0.0007	0.0004	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000
1	0.0067	0.0046	0.0031	0.0021	0.0014	0.0010	0.0007	0.0005
2	0.0327	0.0236	0.0170	0.0121	0.0086	0.0061	0.0043	0.0030
3	0.1028	0.0783	0.0591	0.0444	0.0331	0.0245	0.0181	0.0133
4	0.2348	0.1886	0.1500	0.1182	0.0924	0.0716	0.0551	0.0422
5	0.4197	0.3550	0.2968	0.2454	0.2009	0.1629	0.1309	0.1044
6	0.6188	0.5491	0.4812	0.4166	0.3567	0.3022	0.2534	0.2106
7	0.7872	0.7283	0.6656	0.6010	0.5365	0.4736	0.4136	0.3575
8	0.9006	0.8609	0.8145	0.7624	0.7059	0.6466	0.5860	0.5257
9	0.9617	0.9403	0.9125	0.8782	0.8377	0.7916	0.7408	0.6866
10	0.9880	0.9788	0.9653	0.9468	0.9228	0.8930	0.8575	0.8167
11	0.9970	0.9938	0.9886	0.9804	0.9687	0.9526	0.9318	0.9058
12	0.9994	0.9986	0.9969	0.9940	0.9892	0.9820	0.9717	0.9577
13	0.9999	0.9997	0.9993	0.9985	0.9969	0.9942	0.9900	0.9836
14	1.0000	1.0000	0.9999	0.9997	0.9993	0.9984	0.9970	0.9945
15	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9984
16	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,35$

$x\backslash n$	9	10	11	12	13	14	15	16
0	0.0207	0.0135	0.0088	0.0057	0.0037	0.0024	0.0016	0.0010
1	0.1211	0.0860	0.0606	0.0424	0.0296	0.0205	0.0142	0.0098
2	0.3373	0.2616	0.2001	0.1513	0.1132	0.0839	0.0617	0.0451
3	0.6089	0.5138	0.4256	0.3467	0.2783	0.2205	0.1727	0.1339
4	0.8283	0.7515	0.6683	0.5833	0.5005	0.4227	0.3519	0.2892
5	0.9464	0.9051	0.8513	0.7873	0.7159	0.6405	0.5643	0.4900
6	0.9888	0.9740	0.9499	0.9154	0.8705	0.8164	0.7548	0.6881
7	0.9986	0.9952	0.9878	0.9745	0.9538	0.9247	0.8868	0.8406
8	0.9999	0.9995	0.9980	0.9944	0.9874	0.9757	0.9578	0.9329
9	1.0000	1.0000	0.9998	0.9992	0.9975	0.9940	0.9876	0.9771
10	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9972	0.9938
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9987
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	25	26	27	28	29	30	31	32
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0003	0.0002	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
2	0.0021	0.0015	0.0010	0.0007	0.0005	0.0003	0.0002	0.0002
3	0.0097	0.0070	0.0051	0.0037	0.0026	0.0019	0.0014	0.0010
4	0.0320	0.0242	0.0182	0.0136	0.0101	0.0075	0.0056	0.0041
5	0.0826	0.0649	0.0507	0.0393	0.0303	0.0233	0.0177	0.0135
6	0.1734	0.1416	0.1148	0.0923	0.0738	0.0586	0.0462	0.0362
7	0.3061	0.2596	0.2183	0.1821	0.1507	0.1238	0.1009	0.0818
8	0.4668	0.4106	0.3577	0.3089	0.2645	0.2247	0.1894	0.1584
9	0.6303	0.5731	0.5162	0.4607	0.4076	0.3575	0.3110	0.2685
10	0.7712	0.7219	0.6698	0.6160	0.5617	0.5078	0.4552	0.4047
11	0.8746	0.8384	0.7976	0.7529	0.7050	0.6548	0.6034	0.5515
12	0.9396	0.9168	0.8894	0.8572	0.8207	0.7802	0.7363	0.6898
13	0.9745	0.9623	0.9464	0.9264	0.9022	0.8737	0.8410	0.8043
14	0.9907	0.9850	0.9771	0.9663	0.9524	0.9348	0.9134	0.8881
15	0.9971	0.9948	0.9914	0.9864	0.9794	0.9699	0.9576	0.9422
16	0.9992	0.9985	0.9972	0.9952	0.9921	0.9876	0.9814	0.9731
17	0.9998	0.9996	0.9992	0.9985	0.9973	0.9955	0.9927	0.9888
18	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9975	0.9958
19	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986
20	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,40$

$x\backslash n$	1	2	3	4	5	6	7	8
0	0.6000	0.3600	0.2160	0.1296	0.0778	0.0467	0.0280	0.0168
1	1.0000	0.8400	0.6480	0.4752	0.3370	0.2333	0.1586	0.1064
2	1.0000	1.0000	0.9360	0.8208	0.6826	0.5443	0.4199	0.3154
3	1.0000	1.0000	1.0000	0.9744	0.9130	0.8208	0.7102	0.5941
4	1.0000	1.0000	1.0000	1.0000	0.9898	0.9590	0.9037	0.8263
5	1.0000	1.0000	1.0000	1.0000	1.0000	0.9959	0.9812	0.9502
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9984	0.9915
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9993
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	17	18	19	20	21	22	23	24
0	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0021	0.0013	0.0008	0.0005	0.0003	0.0002	0.0001	0.0001
2	0.0123	0.0082	0.0055	0.0036	0.0024	0.0016	0.0010	0.0007
3	0.0464	0.0328	0.0230	0.0160	0.0110	0.0076	0.0052	0.0035
4	0.1260	0.0942	0.0696	0.0510	0.0370	0.0266	0.0190	0.0134
5	0.2639	0.2088	0.1629	0.1256	0.0957	0.0722	0.0540	0.0400
6	0.4478	0.3743	0.3081	0.2500	0.2002	0.1584	0.1240	0.0960
7	0.6405	0.5634	0.4878	0.4159	0.3495	0.2898	0.2373	0.1919
8	0.8011	0.7368	0.6675	0.5956	0.5237	0.4540	0.3884	0.3279
9	0.9081	0.8653	0.8139	0.7553	0.6914	0.6244	0.5562	0.4891
10	0.9652	0.9424	0.9115	0.8725	0.8256	0.7720	0.7129	0.6502
11	0.9894	0.9797	0.9648	0.9435	0.9151	0.8793	0.8364	0.7870
12	0.9975	0.9942	0.9884	0.9790	0.9648	0.9449	0.9187	0.8857
13	0.9995	0.9987	0.9969	0.9935	0.9877	0.9785	0.9651	0.9465
14	0.9999	0.9998	0.9994	0.9984	0.9964	0.9930	0.9872	0.9783
15	1.0000	1.0000	0.9999	0.9997	0.9992	0.9981	0.9960	0.9925
16	1.0000	1.0000	1.0000	1.0000	0.9998	0.9996	0.9990	0.9978
17	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,40$

$x\backslash n$	9	10	11	12	13	14	15	16
0	0.0101	0.0060	0.0036	0.0022	0.0013	0.0008	0.0005	0.0003
1	0.0705	0.0464	0.0302	0.0196	0.0126	0.0081	0.0052	0.0033
2	0.2318	0.1673	0.1189	0.0834	0.0579	0.0398	0.0271	0.0183
3	0.4826	0.3823	0.2963	0.2253	0.1686	0.1243	0.0905	0.0651
4	0.7334	0.6331	0.5328	0.4382	0.3530	0.2793	0.2173	0.1666
5	0.9006	0.8338	0.7535	0.6652	0.5744	0.4859	0.4032	0.3288
6	0.9750	0.9452	0.9006	0.8418	0.7712	0.6925	0.6098	0.5272
7	0.9962	0.9877	0.9707	0.9427	0.9023	0.8499	0.7869	0.7161
8	0.9997	0.9983	0.9941	0.9847	0.9679	0.9417	0.9050	0.8577
9	1.0000	0.9999	0.9993	0.9972	0.9922	0.9825	0.9662	0.9417
10	1.0000	1.0000	1.0000	0.9997	0.9987	0.9961	0.9907	0.9809
11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9981	0.9951
12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	25	26	27	28	29	30	31	32
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0004	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
3	0.0024	0.0016	0.0011	0.0007	0.0005	0.0003	0.0002	0.0001
4	0.0095	0.0066	0.0046	0.0032	0.0022	0.0015	0.0010	0.0007
5	0.0294	0.0214	0.0155	0.0111	0.0080	0.0057	0.0040	0.0028
6	0.0736	0.0559	0.0421	0.0315	0.0233	0.0172	0.0126	0.0091
7	0.1536	0.1216	0.0953	0.0740	0.0570	0.0435	0.0330	0.0248
8	0.2735	0.2255	0.1839	0.1485	0.1187	0.0940	0.0738	0.0575
9	0.4246	0.3642	0.3087	0.2588	0.2147	0.1763	0.1434	0.1156
10	0.5858	0.5213	0.4585	0.3986	0.3427	0.2915	0.2454	0.2046
11	0.7323	0.6737	0.6127	0.5510	0.4900	0.4311	0.3752	0.3233
12	0.8462	0.8007	0.7499	0.6950	0.6374	0.5785	0.5195	0.4618
13	0.9222	0.8918	0.8553	0.8132	0.7659	0.7145	0.6601	0.6039
14	0.9656	0.9482	0.9257	0.8975	0.8638	0.8246	0.7806	0.7324
15	0.9868	0.9783	0.9663	0.9501	0.9290	0.9029	0.8716	0.8352
16	0.9957	0.9921	0.9866	0.9785	0.9671	0.9519	0.9323	0.9080
17	0.9988	0.9975	0.9954	0.9919	0.9865	0.9788	0.9680	0.9537
18	0.9997	0.9993	0.9986	0.9973	0.9951	0.9917	0.9865	0.9791
19	0.9999	0.9999	0.9997	0.9992	0.9985	0.9971	0.9950	0.9916
20	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9983	0.9970
21	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9991
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,45$

$x\backslash n$	1	2	3	4	5	6	7	8
0	0.5500	0.3025	0.1664	0.0915	0.0503	0.0277	0.0152	0.0084
1	1.0000	0.7975	0.5748	0.3910	0.2562	0.1636	0.1024	0.0632
2	1.0000	1.0000	0.9089	0.7585	0.5931	0.4415	0.3164	0.2201
3	1.0000	1.0000	1.0000	0.9590	0.8688	0.7447	0.6083	0.4770
4	1.0000	1.0000	1.0000	1.0000	0.9815	0.9308	0.8471	0.7396
5	1.0000	1.0000	1.0000	1.0000	1.0000	0.9917	0.9643	0.9115
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9963	0.9819
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9983
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	17	18	19	20	21	22	23	24
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
2	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001
3	0.0184	0.0120	0.0077	0.0049	0.0031	0.0020	0.0012	0.0008
4	0.0596	0.0411	0.0280	0.0189	0.0126	0.0083	0.0055	0.0036
5	0.1471	0.1077	0.0777	0.0553	0.0389	0.0271	0.0186	0.0127
6	0.2902	0.2258	0.1727	0.1299	0.0964	0.0705	0.0510	0.0364
7	0.4743	0.3915	0.3169	0.2520	0.1971	0.1518	0.1152	0.0863
8	0.6626	0.5778	0.4940	0.4143	0.3413	0.2764	0.2203	0.1730
9	0.8166	0.7473	0.6710	0.5914	0.5117	0.4350	0.3636	0.2991
10	0.9174	0.8720	0.8159	0.7507	0.6790	0.6037	0.5278	0.4539
11	0.9699	0.9463	0.9129	0.8692	0.8159	0.7543	0.6865	0.6151
12	0.9914	0.9817	0.9658	0.9420	0.9092	0.8672	0.8164	0.7580
13	0.9981	0.9951	0.9891	0.9786	0.9621	0.9383	0.9063	0.8659
14	0.9997	0.9990	0.9972	0.9936	0.9868	0.9757	0.9589	0.9352
15	1.0000	0.9999	0.9995	0.9985	0.9963	0.9920	0.9847	0.9731
16	1.0000	1.0000	0.9999	0.9997	0.9992	0.9979	0.9952	0.9905
17	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9988	0.9972
18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,45$

$x\backslash n$	9	10	11	12	13	14	15	16
0	0.0046	0.0025	0.0014	0.0008	0.0004	0.0002	0.0001	0.0001
1	0.0385	0.0233	0.0139	0.0083	0.0049	0.0029	0.0017	0.0010
2	0.1495	0.0996	0.0652	0.0421	0.0269	0.0170	0.0107	0.0066
3	0.3614	0.2660	0.1911	0.1345	0.0929	0.0632	0.0424	0.0281
4	0.6214	0.5044	0.3971	0.3044	0.2279	0.1672	0.1204	0.0853
5	0.8342	0.7384	0.6331	0.5269	0.4268	0.3373	0.2608	0.1976
6	0.9502	0.8980	0.8262	0.7393	0.6437	0.5461	0.4522	0.3660
7	0.9909	0.9726	0.9390	0.8883	0.8212	0.7414	0.6535	0.5629
8	0.9992	0.9955	0.9852	0.9644	0.9302	0.8811	0.8182	0.7441
9	1.0000	0.9997	0.9978	0.9921	0.9797	0.9574	0.9231	0.8759
10	1.0000	1.0000	0.9998	0.9989	0.9959	0.9886	0.9745	0.9514
11	1.0000	1.0000	1.0000	0.9999	0.9995	0.9978	0.9937	0.9851
12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9989	0.9965
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	25	26	27	28	29	30	31	32
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0005	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
4	0.0023	0.0015	0.0009	0.0006	0.0004	0.0002	0.0002	0.0001
5	0.0086	0.0058	0.0038	0.0025	0.0017	0.0011	0.0007	0.0005
6	0.0258	0.0180	0.0125	0.0086	0.0059	0.0040	0.0027	0.0018
7	0.0639	0.0467	0.0338	0.0242	0.0172	0.0121	0.0085	0.0059
8	0.1340	0.1024	0.0774	0.0578	0.0427	0.0312	0.0226	0.0162
9	0.2424	0.1936	0.1526	0.1187	0.0913	0.0694	0.0522	0.0389
10	0.3843	0.3204	0.2633	0.2135	0.1708	0.1350	0.1055	0.0815
11	0.5426	0.4713	0.4034	0.3404	0.2833	0.2327	0.1887	0.1513
12	0.6937	0.6257	0.5562	0.4875	0.4213	0.3592	0.3023	0.2512
13	0.8173	0.7617	0.7005	0.6356	0.5689	0.5025	0.4380	0.3769
14	0.9040	0.8650	0.8185	0.7654	0.7070	0.6448	0.5808	0.5165
15	0.9560	0.9326	0.9022	0.8645	0.8199	0.7691	0.7132	0.6536
16	0.9826	0.9707	0.9536	0.9304	0.9008	0.8644	0.8215	0.7728
17	0.9942	0.9890	0.9807	0.9685	0.9514	0.9286	0.8997	0.8645
18	0.9984	0.9965	0.9931	0.9875	0.9790	0.9666	0.9495	0.9271
19	0.9996	0.9991	0.9979	0.9957	0.9920	0.9862	0.9773	0.9648
20	0.9999	0.9998	0.9995	0.9988	0.9974	0.9950	0.9910	0.9849
21	1.0000	1.0000	0.9999	0.9997	0.9993	0.9984	0.9969	0.9942
22	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9981
23	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,50$

$x\backslash n$	1	2	3	4	5	6	7	8
0	0.5000	0.2500	0.1250	0.0625	0.0312	0.0156	0.0078	0.0039
1	1.0000	0.7500	0.5000	0.3125	0.1875	0.1094	0.0625	0.0352
2	1.0000	1.0000	0.8750	0.6875	0.5000	0.3437	0.2266	0.1445
3	1.0000	1.0000	1.0000	0.9375	0.8125	0.6562	0.5000	0.3633
4	1.0000	1.0000	1.0000	1.0000	0.9688	0.8906	0.7734	0.6367
5	1.0000	1.0000	1.0000	1.0000	1.0000	0.9844	0.9375	0.8555
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9922	0.9648
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9961
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	17	18	19	20	21	22	23	24
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0012	0.0007	0.0004	0.0002	0.0001	0.0001	0.0000	0.0000
3	0.0064	0.0038	0.0022	0.0013	0.0007	0.0004	0.0002	0.0001
4	0.0245	0.0154	0.0096	0.0059	0.0036	0.0022	0.0013	0.0008
5	0.0717	0.0481	0.0318	0.0207	0.0133	0.0085	0.0053	0.0033
6	0.1662	0.1189	0.0835	0.0577	0.0392	0.0262	0.0173	0.0113
7	0.3145	0.2403	0.1796	0.1316	0.0946	0.0669	0.0466	0.0320
8	0.5000	0.4073	0.3238	0.2517	0.1917	0.1431	0.1050	0.0758
9	0.6855	0.5927	0.5000	0.4119	0.3318	0.2617	0.2024	0.1537
10	0.8338	0.7597	0.6762	0.5881	0.5000	0.4159	0.3388	0.2706
11	0.9283	0.8811	0.8204	0.7483	0.6682	0.5841	0.5000	0.4194
12	0.9755	0.9519	0.9165	0.8684	0.8083	0.7383	0.6612	0.5806
13	0.9936	0.9846	0.9682	0.9423	0.9054	0.8569	0.7976	0.7294
14	0.9988	0.9962	0.9904	0.9793	0.9608	0.9331	0.8950	0.8463
15	0.9999	0.9993	0.9978	0.9941	0.9867	0.9738	0.9534	0.9242
16	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9827	0.9680
17	1.0000	1.0000	1.0000	0.9998	0.9993	0.9978	0.9947	0.9887
18	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9967
19	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Binomialverteilung für $p = 0,50$

$x\backslash n$	9	10	11	12	13	14	15	16
0	0.0020	0.0010	0.0005	0.0002	0.0001	0.0001	0.0000	0.0000
1	0.0195	0.0107	0.0059	0.0032	0.0017	0.0009	0.0005	0.0003
2	0.0898	0.0547	0.0327	0.0193	0.0112	0.0065	0.0037	0.0021
3	0.2539	0.1719	0.1133	0.0730	0.0461	0.0287	0.0176	0.0106
4	0.5000	0.3770	0.2744	0.1938	0.1334	0.0898	0.0592	0.0384
5	0.7461	0.6230	0.5000	0.3872	0.2905	0.2120	0.1509	0.1051
6	0.9102	0.8281	0.7256	0.6128	0.5000	0.3953	0.3036	0.2272
7	0.9805	0.9453	0.8867	0.8062	0.7095	0.6047	0.5000	0.4018
8	0.9980	0.9893	0.9673	0.9270	0.8666	0.7880	0.6964	0.5982
9	1.0000	0.9990	0.9941	0.9807	0.9539	0.9102	0.8491	0.7728
10	1.0000	1.0000	0.9995	0.9968	0.9888	0.9713	0.9408	0.8949
11	1.0000	1.0000	1.0000	0.9998	0.9983	0.9935	0.9824	0.9616
12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9963	0.9894
13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9979
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash n$	25	26	27	28	29	30	31	32
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0005	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
5	0.0020	0.0012	0.0008	0.0005	0.0003	0.0002	0.0001	0.0001
6	0.0073	0.0047	0.0030	0.0019	0.0012	0.0007	0.0004	0.0003
7	0.0216	0.0145	0.0096	0.0063	0.0041	0.0026	0.0017	0.0011
8	0.0539	0.0378	0.0261	0.0178	0.0121	0.0081	0.0053	0.0035
9	0.1148	0.0843	0.0610	0.0436	0.0307	0.0214	0.0147	0.0100
10	0.2122	0.1635	0.1239	0.0925	0.0680	0.0494	0.0354	0.0251
11	0.3450	0.2786	0.2210	0.1725	0.1325	0.1002	0.0748	0.0551
12	0.5000	0.4225	0.3506	0.2858	0.2291	0.1808	0.1405	0.1077
13	0.6550	0.5775	0.5000	0.4253	0.3555	0.2923	0.2366	0.1885
14	0.7878	0.7214	0.6494	0.5747	0.5000	0.4278	0.3601	0.2983
15	0.8852	0.8365	0.7790	0.7142	0.6445	0.5722	0.5000	0.4300
16	0.9461	0.9157	0.8761	0.8275	0.7709	0.7077	0.6399	0.5700
17	0.9784	0.9622	0.9390	0.9075	0.8675	0.8192	0.7634	0.7017
18	0.9927	0.9855	0.9739	0.9564	0.9320	0.8998	0.8595	0.8115
19	0.9980	0.9953	0.9904	0.9822	0.9693	0.9506	0.9252	0.8923
20	0.9995	0.9988	0.9970	0.9937	0.9879	0.9786	0.9646	0.9449
21	0.9999	0.9997	0.9992	0.9981	0.9959	0.9919	0.9853	0.9749
22	1.0000	1.0000	0.9998	0.9995	0.9988	0.9974	0.9947	0.9900
23	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9983	0.9965
24	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9989
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

14.2 Verteilungsfunktion $F(x)$ der Poissonverteilung ($\lambda = 0,1 \ldots 3,0$)

$x \backslash \lambda$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x \backslash \lambda$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337	0.4060
2	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037	0.6767
3	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747	0.8571
4	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559	0.9473
5	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868	0.9834
6	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966	0.9955
7	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992	0.9989
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9998
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x \backslash \lambda$	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
0	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550	0.0498
1	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146	0.1991
2	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460	0.4232
3	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696	0.6472
4	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318	0.8153
5	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258	0.9161
6	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9794	0.9756	0.9713	0.9665
7	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901	0.9881
8	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969	0.9962
9	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991	0.9989
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Poissonverteilung ($\lambda = 3,1 \ldots 5,0$)

$x \backslash \lambda$	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
0	0.0450	0.0408	0.0369	0.0334	0.0302	0.0273	0.0247	0.0224	0.0202	0.0183
1	0.1847	0.1712	0.1586	0.1468	0.1359	0.1257	0.1162	0.1074	0.0992	0.0916
2	0.4012	0.3799	0.3594	0.3397	0.3208	0.3027	0.2854	0.2689	0.2531	0.2381
3	0.6248	0.6025	0.5803	0.5584	0.5366	0.5152	0.4942	0.4735	0.4532	0.4335
4	0.7982	0.7806	0.7626	0.7442	0.7254	0.7064	0.6872	0.6678	0.6484	0.6288
5	0.9057	0.8946	0.8829	0.8705	0.8576	0.8441	0.8301	0.8156	0.8006	0.7851
6	0.9612	0.9554	0.9490	0.9421	0.9347	0.9267	0.9182	0.9091	0.8995	0.8893
7	0.9858	0.9832	0.9802	0.9769	0.9733	0.9692	0.9648	0.9599	0.9546	0.9489
8	0.9953	0.9943	0.9931	0.9917	0.9901	0.9883	0.9863	0.9840	0.9815	0.9786
9	0.9986	0.9982	0.9978	0.9973	0.9967	0.9960	0.9952	0.9942	0.9931	0.9919
10	0.9996	0.9995	0.9994	0.9992	0.9990	0.9987	0.9984	0.9981	0.9977	0.9972
11	0.9999	0.9999	0.9998	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	0.9991
12	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x \backslash \lambda$	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5
0	0.0166	0.0150	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074	0.0067
1	0.0845	0.0780	0.0719	0.0663	0.0611	0.0563	0.0518	0.0477	0.0439	0.0404
2	0.2238	0.2102	0.1974	0.1851	0.1736	0.1626	0.1523	0.1425	0.1333	0.1247
3	0.4142	0.3954	0.3772	0.3594	0.3423	0.3257	0.3097	0.2942	0.2793	0.2650
4	0.6093	0.5898	0.5704	0.5512	0.5321	0.5132	0.4946	0.4763	0.4582	0.4405
5	0.7693	0.7531	0.7367	0.7199	0.7029	0.6858	0.6684	0.6510	0.6335	0.6160
6	0.8786	0.8675	0.8558	0.8436	0.8311	0.8180	0.8046	0.7908	0.7767	0.7622
7	0.9427	0.9361	0.9290	0.9214	0.9134	0.9049	0.8960	0.8867	0.8769	0.8666
8	0.9755	0.9721	0.9683	0.9642	0.9597	0.9549	0.9497	0.9442	0.9382	0.9319
9	0.9905	0.9889	0.9871	0.9851	0.9829	0.9805	0.9778	0.9749	0.9717	0.9682
10	0.9966	0.9959	0.9952	0.9943	0.9933	0.9922	0.9910	0.9896	0.9880	0.9863
11	0.9989	0.9986	0.9983	0.9980	0.9976	0.9971	0.9966	0.9960	0.9953	0.9945
12	0.9997	0.9996	0.9995	0.9993	0.9992	0.9990	0.9988	0.9986	0.9983	0.9980
13	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994	0.9993
14	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Poissonverteilung ($\lambda = 4,1 \ldots 7,0$)

$x\backslash\lambda$	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6
0	0.0061	0.0055	0.0050	0.0045	0.0041	0.0037	0.0033	0.0030	0.0027	0.0025
1	0.0372	0.0342	0.0314	0.0289	0.0266	0.0244	0.0224	0.0206	0.0189	0.0174
2	0.1165	0.1088	0.1016	0.0948	0.0884	0.0824	0.0768	0.0715	0.0666	0.0620
3	0.2513	0.2381	0.2254	0.2133	0.2017	0.1906	0.1800	0.1700	0.1604	0.1512
4	0.4231	0.4061	0.3895	0.3733	0.3575	0.3422	0.3272	0.3127	0.2987	0.2851
5	0.5984	0.5809	0.5635	0.5461	0.5289	0.5119	0.4950	0.4783	0.4619	0.4457
6	0.7474	0.7324	0.7171	0.7017	0.6860	0.6703	0.6544	0.6384	0.6224	0.6063
7	0.8560	0.8449	0.8335	0.8217	0.8095	0.7970	0.7841	0.7710	0.7576	0.7440
8	0.9252	0.9181	0.9106	0.9027	0.8944	0.8857	0.8766	0.8672	0.8574	0.8472
9	0.9644	0.9603	0.9559	0.9512	0.9462	0.9409	0.9352	0.9292	0.9228	0.9161
10	0.9844	0.9823	0.9800	0.9775	0.9747	0.9718	0.9686	0.9651	0.9614	0.9574
11	0.9937	0.9927	0.9916	0.9904	0.9890	0.9875	0.9859	0.9841	0.9821	0.9799
12	0.9976	0.9972	0.9967	0.9962	0.9955	0.9949	0.9941	0.9932	0.9922	0.9912
13	0.9992	0.9990	0.9988	0.9986	0.9983	0.9980	0.9977	0.9973	0.9969	0.9964
14	0.9997	0.9997	0.9996	0.9995	0.9994	0.9993	0.9991	0.9990	0.9988	0.9986
15	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997	0.9996	0.9996	0.9995
16	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x\backslash\lambda$	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7
0	0.0022	0.0020	0.0018	0.0017	0.0015	0.0014	0.0012	0.0011	0.0010	0.0009
1	0.0159	0.0146	0.0134	0.0123	0.0113	0.0103	0.0095	0.0087	0.0080	0.0073
2	0.0577	0.0536	0.0498	0.0463	0.0430	0.0400	0.0371	0.0344	0.0320	0.0296
3	0.1425	0.1342	0.1264	0.1189	0.1118	0.1052	0.0988	0.0928	0.0871	0.0818
4	0.2719	0.2592	0.2469	0.2351	0.2237	0.2127	0.2022	0.1920	0.1823	0.1730
5	0.4298	0.4141	0.3988	0.3837	0.3690	0.3547	0.3406	0.3270	0.3137	0.3007
6	0.5902	0.5742	0.5582	0.5423	0.5265	0.5108	0.4953	0.4799	0.4647	0.4497
7	0.7301	0.7160	0.7017	0.6873	0.6728	0.6581	0.6433	0.6285	0.6136	0.5987
8	0.8367	0.8259	0.8148	0.8033	0.7916	0.7796	0.7673	0.7548	0.7420	0.7291
9	0.9090	0.9016	0.8939	0.8858	0.8774	0.8686	0.8596	0.8502	0.8405	0.8305
10	0.9531	0.9486	0.9437	0.9386	0.9332	0.9274	0.9214	0.9151	0.9084	0.9015
11	0.9776	0.9750	0.9723	0.9693	0.9661	0.9627	0.9591	0.9552	0.9510	0.9467
12	0.9900	0.9887	0.9873	0.9857	0.9840	0.9821	0.9801	0.9779	0.9755	0.9730
13	0.9958	0.9952	0.9945	0.9937	0.9929	0.9920	0.9909	0.9898	0.9885	0.9872
14	0.9984	0.9981	0.9978	0.9974	0.9970	0.9966	0.9961	0.9956	0.9950	0.9943
15	0.9994	0.9993	0.9992	0.9990	0.9988	0.9986	0.9984	0.9982	0.9979	0.9976
16	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9994	0.9993	0.9992	0.9990
17	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996
18	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Poissonverteilung ($\lambda = 7,1 \ldots 8,0$)

$x\backslash\lambda$	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8
0	0.0008	0.0007	0.0007	0.0006	0.0006	0.0005	0.0005	0.0004	0.0004	0.0003
1	0.0067	0.0061	0.0056	0.0051	0.0047	0.0043	0.0039	0.0036	0.0033	0.0030
2	0.0275	0.0255	0.0236	0.0219	0.0203	0.0188	0.0174	0.0161	0.0149	0.0138
3	0.0767	0.0719	0.0674	0.0632	0.0591	0.0554	0.0518	0.0485	0.0453	0.0424
4	0.1641	0.1555	0.1473	0.1395	0.1321	0.1249	0.1181	0.1117	0.1055	0.0996
5	0.2881	0.2759	0.2640	0.2526	0.2414	0.2307	0.2203	0.2103	0.2006	0.1912
6	0.4349	0.4204	0.4060	0.3920	0.3782	0.3646	0.3514	0.3384	0.3257	0.3134
7	0.5838	0.5689	0.5541	0.5393	0.5246	0.5100	0.4956	0.4812	0.4670	0.4530
8	0.7160	0.7027	0.6892	0.6757	0.6620	0.6482	0.6343	0.6204	0.6065	0.5925
9	0.8202	0.8096	0.7988	0.7877	0.7764	0.7649	0.7531	0.7411	0.7290	0.7166
10	0.8942	0.8867	0.8788	0.8707	0.8622	0.8535	0.8445	0.8352	0.8257	0.8159
11	0.9420	0.9371	0.9319	0.9265	0.9208	0.9148	0.9085	0.9020	0.8952	0.8881
12	0.9703	0.9673	0.9642	0.9609	0.9573	0.9536	0.9496	0.9454	0.9409	0.9362
13	0.9857	0.9841	0.9824	0.9805	0.9784	0.9762	0.9739	0.9714	0.9687	0.9658
14	0.9935	0.9927	0.9918	0.9908	0.9897	0.9886	0.9873	0.9859	0.9844	0.9827
15	0.9972	0.9969	0.9964	0.9959	0.9954	0.9948	0.9941	0.9934	0.9926	0.9918
16	0.9989	0.9987	0.9985	0.9983	0.9980	0.9978	0.9974	0.9971	0.9967	0.9963
17	0.9996	0.9995	0.9994	0.9993	0.9992	0.9991	0.9989	0.9988	0.9986	0.9984
18	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9994	0.9993
19	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Beispiel: Die Zufallsvariable $X \sim Po(7,5)$ und gesucht ist

$$\begin{aligned} P(X = 4) &= F(4) - F(3) = 0,1321 - 0,0591 = 0,0730 \\ P(2 \leq X \leq 6) &= F(6) - F(1) = 0,3782 - 0,0047 = 0,3735 \\ P(X > 6) &= 1 - F(6) = 1 - 0,3782 = 0,6218 \end{aligned}$$

Verteilungsfunktion $F(x)$ der Poissonverteilung ($\lambda = 8,1 \dots 9,0$)

$x\backslash\lambda$	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9
0	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001
1	0.0028	0.0025	0.0023	0.0021	0.0019	0.0018	0.0016	0.0015	0.0014	0.0012
2	0.0127	0.0118	0.0109	0.0100	0.0093	0.0086	0.0079	0.0073	0.0068	0.0062
3	0.0396	0.0370	0.0346	0.0323	0.0301	0.0281	0.0262	0.0244	0.0228	0.0212
4	0.0940	0.0887	0.0837	0.0789	0.0744	0.0701	0.0660	0.0621	0.0584	0.0550
5	0.1822	0.1736	0.1653	0.1573	0.1496	0.1422	0.1352	0.1284	0.1219	0.1157
6	0.3013	0.2896	0.2781	0.2670	0.2562	0.2457	0.2355	0.2256	0.2160	0.2068
7	0.4391	0.4254	0.4119	0.3987	0.3856	0.3728	0.3602	0.3478	0.3357	0.3239
8	0.5786	0.5647	0.5507	0.5369	0.5231	0.5094	0.4958	0.4823	0.4689	0.4557
9	0.7041	0.6915	0.6788	0.6659	0.6530	0.6400	0.6269	0.6137	0.6006	0.5874
10	0.8058	0.7955	0.7850	0.7743	0.7634	0.7522	0.7409	0.7294	0.7178	0.7060
11	0.8807	0.8731	0.8652	0.8571	0.8487	0.8400	0.8311	0.8220	0.8126	0.8030
12	0.9313	0.9261	0.9207	0.9150	0.9091	0.9029	0.8965	0.8898	0.8829	0.8758
13	0.9628	0.9595	0.9561	0.9524	0.9486	0.9445	0.9403	0.9358	0.9311	0.9261
14	0.9810	0.9791	0.9771	0.9749	0.9726	0.9701	0.9675	0.9647	0.9617	0.9585
15	0.9908	0.9898	0.9887	0.9875	0.9862	0.9848	0.9832	0.9816	0.9798	0.9780
16	0.9958	0.9953	0.9947	0.9941	0.9934	0.9926	0.9918	0.9909	0.9899	0.9889
17	0.9982	0.9979	0.9977	0.9973	0.9970	0.9966	0.9962	0.9957	0.9952	0.9947
18	0.9992	0.9991	0.9990	0.9989	0.9987	0.9985	0.9983	0.9981	0.9978	0.9976
19	0.9997	0.9997	0.9996	0.9995	0.9995	0.9994	0.9993	0.9992	0.9991	0.9989
20	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996
21	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Verteilungsfunktion $F(x)$ der Poissonverteilung ($\lambda = 9,1 \dots 10$)

$x\backslash\lambda$	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000
1	0.0011	0.0010	0.0009	0.0009	0.0008	0.0007	0.0007	0.0006	0.0005	0.0005
2	0.0058	0.0053	0.0049	0.0045	0.0042	0.0038	0.0035	0.0033	0.0030	0.0028
3	0.0198	0.0184	0.0172	0.0160	0.0149	0.0138	0.0129	0.0120	0.0111	0.0103
4	0.0517	0.0486	0.0456	0.0429	0.0403	0.0378	0.0355	0.0333	0.0312	0.0293
5	0.1098	0.1041	0.0986	0.0935	0.0885	0.0838	0.0793	0.0750	0.0710	0.0671
6	0.1978	0.1892	0.1808	0.1727	0.1649	0.1574	0.1502	0.1433	0.1366	0.1301
7	0.3123	0.3010	0.2900	0.2792	0.2687	0.2584	0.2485	0.2388	0.2294	0.2202
8	0.4426	0.4296	0.4168	0.4042	0.3918	0.3796	0.3676	0.3558	0.3442	0.3328
9	0.5742	0.5611	0.5479	0.5349	0.5218	0.5089	0.4960	0.4832	0.4705	0.4579
10	0.6941	0.6820	0.6699	0.6576	0.6453	0.6329	0.6205	0.6080	0.5955	0.5830
11	0.7932	0.7832	0.7730	0.7626	0.7520	0.7412	0.7303	0.7193	0.7081	0.6968
12	0.8684	0.8607	0.8529	0.8448	0.8364	0.8279	0.8191	0.8101	0.8009	0.7916
13	0.9210	0.9156	0.9100	0.9042	0.8981	0.8919	0.8853	0.8786	0.8716	0.8645
14	0.9552	0.9517	0.9480	0.9441	0.9400	0.9357	0.9312	0.9265	0.9216	0.9165
15	0.9760	0.9738	0.9715	0.9691	0.9665	0.9638	0.9609	0.9579	0.9546	0.9513
16	0.9878	0.9865	0.9852	0.9838	0.9823	0.9806	0.9789	0.9770	0.9751	0.9730
17	0.9941	0.9934	0.9927	0.9919	0.9911	0.9902	0.9892	0.9881	0.9870	0.9857
18	0.9973	0.9969	0.9966	0.9962	0.9957	0.9952	0.9947	0.9941	0.9935	0.9928
19	0.9988	0.9986	0.9985	0.9983	0.9980	0.9978	0.9975	0.9972	0.9969	0.9965
20	0.9995	0.9994	0.9993	0.9992	0.9991	0.9990	0.9989	0.9987	0.9986	0.9984
21	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995	0.9994	0.9993
22	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997	0.9997
23	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

14.3 Quantile x_p der χ^2-Verteilung mit f Freiheitsgraden

$p\backslash f$	1	2	3	4	5	6	7	8	9	10
0.005	0.00	0.01	0.07	0.21	0.41	0.68	0.99	1.34	1.73	2.16
0.01	0.00	0.02	0.11	0.30	0.55	0.87	1.24	1.65	2.09	2.56
0.025	0.00	0.05	0.22	0.48	0.83	1.24	1.69	2.18	2.70	3.25
0.05	0.00	0.10	0.35	0.71	1.15	1.64	2.17	2.73	3.33	3.94
0.1	0.02	0.21	0.58	1.06	1.61	2.20	2.83	3.49	4.17	4.87
0.2	0.06	0.45	1.01	1.65	2.34	3.07	3.82	4.59	5.38	6.18
0.25	0.10	0.58	1.21	1.92	2.67	3.45	4.25	5.07	5.90	6.74
0.3	0.15	0.71	1.42	2.19	3.00	3.83	4.67	5.53	6.39	7.27
0.4	0.27	1.02	1.87	2.75	3.66	4.57	5.49	6.42	7.36	8.30
0.5	0.45	1.39	2.37	3.36	4.35	5.35	6.35	7.34	8.34	9.34
0.6	0.71	1.83	2.95	4.04	5.13	6.21	7.28	8.35	9.41	10.47
0.7	1.07	2.41	3.66	4.88	6.06	7.23	8.38	9.52	10.66	11.78
0.75	1.32	2.77	4.11	5.39	6.63	7.84	9.04	10.22	11.39	12.55
0.8	1.64	3.22	4.64	5.99	7.29	8.56	9.80	11.03	12.24	13.44
0.9	2.71	4.61	6.25	7.78	9.24	10.64	12.02	13.36	14.68	15.99
0.95	3.84	5.99	7.81	9.49	11.07	12.59	14.07	15.51	16.92	18.31
0.975	5.02	7.38	9.35	11.14	12.83	14.45	16.01	17.53	19.02	20.48
0.99	6.63	9.21	11.34	13.28	15.09	16.81	18.48	20.09	21.67	23.21
0.995	7.88	10.60	12.84	14.86	16.75	18.55	20.28	21.95	23.59	25.19

$p\backslash f$	11	12	13	14	15	16	17	18	19	20
0.005	2.60	3.07	3.57	4.07	4.60	5.14	5.70	6.26	6.84	7.43
0.01	3.05	3.57	4.11	4.66	5.23	5.81	6.41	7.01	7.63	8.26
0.025	3.82	4.40	5.01	5.63	6.26	6.91	7.56	8.23	8.91	9.59
0.05	4.57	5.23	5.89	6.57	7.26	7.96	8.67	9.39	10.12	10.85
0.1	5.58	6.30	7.04	7.79	8.55	9.31	10.09	10.86	11.65	12.44
0.2	6.99	7.81	8.63	9.47	10.31	11.15	12.00	12.86	13.72	14.58
0.25	7.58	8.44	9.30	10.17	11.04	11.91	12.79	13.68	14.56	15.45
0.3	8.15	9.03	9.93	10.82	11.72	12.62	13.53	14.44	15.35	16.27
0.4	9.24	10.18	11.13	12.08	13.03	13.98	14.94	15.89	16.85	17.81
0.5	10.34	11.34	12.34	13.34	14.34	15.34	16.34	17.34	18.34	19.34
0.6	11.53	12.58	13.64	14.69	15.73	16.78	17.82	18.87	19.91	20.95
0.7	12.90	14.01	15.12	16.22	17.32	18.42	19.51	20.60	21.69	22.77
0.75	13.70	14.85	15.98	17.12	18.25	19.37	20.49	21.60	22.72	23.83
0.8	14.63	15.81	16.98	18.15	19.31	20.47	21.61	22.76	23.90	25.04
0.9	17.28	18.55	19.81	21.06	22.31	23.54	24.77	25.99	27.20	28.41
0.95	19.68	21.03	22.36	23.68	25.00	26.30	27.59	28.87	30.14	31.41
0.975	21.92	23.34	24.74	26.12	27.49	28.85	30.19	31.53	32.85	34.17
0.99	24.72	26.22	27.69	29.14	30.58	32.00	33.41	34.81	36.19	37.57
0.995	26.76	28.30	29.82	31.32	32.80	34.27	35.72	37.16	38.58	40.00

Quantile x_p der χ^2-Verteilung mit f Freiheitsgraden

$p\backslash f$	21	22	23	24	25	26	27	28	29	30
0.005	8.03	8.64	9.26	9.89	10.52	11.16	11.81	12.46	13.12	13.79
0.01	8.90	9.54	10.20	10.86	11.52	12.20	12.88	13.56	14.26	14.95
0.025	10.28	10.98	11.69	12.40	13.12	13.84	14.57	15.31	16.05	16.79
0.05	11.59	12.34	13.09	13.85	14.61	15.38	16.15	16.93	17.71	18.49
0.1	13.24	14.04	14.85	15.66	16.47	17.29	18.11	18.94	19.77	20.60
0.2	15.44	16.31	17.19	18.06	18.94	19.82	20.70	21.59	22.48	23.36
0.25	16.34	17.24	18.14	19.04	19.94	20.84	21.75	22.66	23.57	24.48
0.3	17.18	18.10	19.02	19.94	20.87	21.79	22.72	23.65	24.58	25.51
0.4	18.77	19.73	20.69	21.65	22.62	23.58	24.54	25.51	26.48	27.44
0.5	20.34	21.34	22.34	23.34	24.34	25.34	26.34	27.34	28.34	29.34
0.6	21.99	23.03	24.07	25.11	26.14	27.18	28.21	29.25	30.28	31.32
0.7	23.86	24.94	26.02	27.10	28.17	29.25	30.32	31.39	32.46	33.53
0.75	24.93	26.04	27.14	28.24	29.34	30.43	31.53	32.62	33.71	34.80
0.8	26.17	27.30	28.43	29.55	30.68	31.79	32.91	34.03	35.14	36.25
0.9	29.62	30.81	32.01	33.20	34.38	35.56	36.74	37.92	39.09	40.26
0.95	32.67	33.92	35.17	36.42	37.65	38.89	40.11	41.34	42.56	43.77
0.975	35.48	36.78	38.08	39.36	40.65	41.92	43.19	44.46	45.72	46.98
0.99	38.93	40.29	41.64	42.98	44.31	45.64	46.96	48.28	49.59	50.89
0.995	41.40	42.80	44.18	45.56	46.93	48.29	49.64	50.99	52.34	53.67

$p\backslash f$	31	32	33	34	35	36	37	38	39	40
0.005	14.46	15.13	15.82	16.50	17.19	17.89	18.59	19.29	20.00	20.71
0.01	15.66	16.36	17.07	17.79	18.51	19.23	19.96	20.69	21.43	22.16
0.025	17.54	18.29	19.05	19.81	20.57	21.34	22.11	22.88	23.65	24.43
0.05	19.28	20.07	20.87	21.66	22.47	23.27	24.07	24.88	25.70	26.51
0.1	21.43	22.27	23.11	23.95	24.80	25.64	26.49	27.34	28.20	29.05
0.2	24.26	25.15	26.04	26.94	27.84	28.73	29.64	30.54	31.44	32.34
0.25	25.39	26.30	27.22	28.14	29.05	29.97	30.89	31.81	32.74	33.66
0.3	26.44	27.37	28.31	29.24	30.18	31.12	32.05	32.99	33.93	34.87
0.4	28.41	29.38	30.34	31.31	32.28	33.25	34.22	35.19	36.16	37.13
0.5	30.34	31.34	32.34	33.34	34.34	35.34	36.34	37.34	38.34	39.34
0.6	32.35	33.38	34.41	35.44	36.47	37.50	38.53	39.56	40.59	41.62
0.7	34.60	35.66	36.73	37.80	38.86	39.92	40.98	42.05	43.11	44.16
0.75	35.89	36.97	38.06	39.14	40.22	41.30	42.38	43.46	44.54	45.62
0.8	37.36	38.47	39.57	40.68	41.78	42.88	43.98	45.08	46.17	47.27
0.9	41.42	42.58	43.75	44.90	46.06	47.21	48.36	49.51	50.66	51.81
0.95	44.99	46.19	47.40	48.60	49.80	51.00	52.19	53.38	54.57	55.76
0.975	48.23	49.48	50.73	51.97	53.20	54.44	55.67	56.90	58.12	59.34
0.99	52.19	53.49	54.78	56.06	57.34	58.62	59.89	61.16	62.43	63.69
0.995	55.00	56.33	57.65	58.96	60.27	61.58	62.88	64.18	65.48	66.77

Beispiel: Die Zufallsvariable $X \sim \chi^2_{27}$ und gesucht ist das 95%-Quantil ($p = 0,95$)

$$F(x_{0,95}) = 0,95 \Longrightarrow x_{0,95} = 40,11$$

14.4 Quantile x_p der t-Verteilung mit f Freiheitsgraden

$p \backslash f$	1	2	3	4	5	6	7	8	9	10
0.6	0.325	0.289	0.277	0.271	0.267	0.265	0.263	0.262	0.261	0.260
0.75	1.000	0.816	0.765	0.741	0.727	0.718	0.711	0.706	0.703	0.700
0.8	1.376	1.061	0.978	0.941	0.920	0.906	0.896	0.889	0.883	0.879
0.9	3.078	1.886	1.638	1.533	1.476	1.440	1.415	1.397	1.383	1.372
0.95	6.314	2.920	2.353	2.132	2.015	1.943	1.895	1.860	1.833	1.812
0.975	12.706	4.303	3.182	2.776	2.571	2.447	2.365	2.306	2.262	2.228
0.99	31.821	6.965	4.541	3.747	3.365	3.143	2.998	2.896	2.821	2.764
0.995	63.657	9.925	5.841	4.604	4.032	3.707	3.499	3.355	3.250	3.169

$p \backslash f$	11	12	13	14	15	16	17	18	19	20
0.6	0.260	0.259	0.259	0.258	0.258	0.258	0.257	0.257	0.257	0.257
0.75	0.697	0.695	0.694	0.692	0.691	0.690	0.689	0.688	0.688	0.687
0.8	0.876	0.873	0.870	0.868	0.866	0.865	0.863	0.862	0.861	0.860
0.9	1.363	1.356	1.350	1.345	1.341	1.337	1.333	1.330	1.328	1.325
0.95	1.796	1.782	1.771	1.761	1.753	1.746	1.740	1.734	1.729	1.725
0.975	2.201	2.179	2.160	2.145	2.131	2.120	2.110	2.101	2.093	2.086
0.99	2.718	2.681	2.650	2.624	2.602	2.583	2.567	2.552	2.539	2.528
0.995	3.106	3.055	3.012	2.977	2.947	2.921	2.898	2.878	2.861	2.845

$p \backslash f$	21	22	23	24	25	26	27	28	29	30
0.6	0.257	0.256	0.256	0.256	0.256	0.256	0.256	0.256	0.256	0.256
0.75	0.686	0.686	0.685	0.685	0.684	0.684	0.684	0.683	0.683	0.683
0.8	0.859	0.858	0.858	0.857	0.856	0.856	0.855	0.855	0.854	0.854
0.9	1.323	1.321	1.319	1.318	1.316	1.315	1.314	1.313	1.311	1.310
0.95	1.721	1.717	1.714	1.711	1.708	1.706	1.703	1.701	1.699	1.697
0.975	2.080	2.074	2.069	2.064	2.060	2.056	2.052	2.048	2.045	2.042
0.99	2.518	2.508	2.500	2.492	2.485	2.479	2.473	2.467	2.462	2.457
0.995	2.831	2.819	2.807	2.797	2.787	2.779	2.771	2.763	2.756	2.750

Beispiel: Die Zufallsvariable $X \sim t_{27}$ und gesucht ist das 95% Quantil ($p = 0,95$)

$$F(x_{0,95}) = 0,95 \Longrightarrow x_{0,95} = 1,703$$

14.5 95% Quantil $x_{0,95}$ der F-Verteilung mit f_1 und f_2 Freiheitsgraden

$f_1 \backslash f_2$	1	2	3	4	5	6	7	8	9
1	161.45	18.51	10.13	7.71	6.61	5.99	5.59	5.32	5.12
2	199.50	19.00	9.55	6.94	5.79	5.14	4.74	4.46	4.26
3	215.71	19.16	9.28	6.59	5.41	4.76	4.35	4.07	3.86
4	224.58	19.25	9.12	6.39	5.19	4.53	4.12	3.84	3.63
5	230.16	19.30	9.01	6.26	5.05	4.39	3.97	3.69	3.48
6	233.99	19.33	8.94	6.16	4.95	4.28	3.87	3.58	3.37
7	236.77	19.35	8.89	6.09	4.88	4.21	3.79	3.50	3.29
8	238.88	19.37	8.85	6.04	4.82	4.15	3.73	3.44	3.23
9	240.54	19.38	8.81	6.00	4.77	4.10	3.68	3.39	3.18
10	241.88	19.40	8.79	5.96	4.74	4.06	3.64	3.35	3.14
15	245.95	19.43	8.70	5.86	4.62	3.94	3.51	3.22	3.01
20	248.01	19.45	8.66	5.80	4.56	3.87	3.44	3.15	2.94
25	249.26	19.46	8.63	5.77	4.52	3.83	3.40	3.11	2.89
30	250.10	19.46	8.62	5.75	4.50	3.81	3.38	3.08	2.86
40	251.14	19.47	8.59	5.72	4.46	3.77	3.34	3.04	2.83
50	251.77	19.48	8.58	5.70	4.44	3.75	3.32	3.02	2.80
75	252.62	19.48	8.56	5.68	4.42	3.73	3.29	2.99	2.77
100	253.04	19.49	8.55	5.66	4.41	3.71	3.27	2.97	2.76

$f_1 \backslash f_2$	10	15	20	25	30	40	50	75	100
1	4.96	4.54	4.35	4.24	4.17	4.08	4.03	3.97	3.94
2	4.10	3.68	3.49	3.39	3.32	3.23	3.18	3.12	3.09
3	3.71	3.29	3.10	2.99	2.92	2.84	2.79	2.73	2.70
4	3.48	3.06	2.87	2.76	2.69	2.61	2.56	2.49	2.46
5	3.33	2.90	2.71	2.60	2.53	2.45	2.40	2.34	2.31
6	3.22	2.79	2.60	2.49	2.42	2.34	2.29	2.22	2.19
7	3.14	2.71	2.51	2.40	2.33	2.25	2.20	2.13	2.10
8	3.07	2.64	2.45	2.34	2.27	2.18	2.13	2.06	2.03
9	3.02	2.59	2.39	2.28	2.21	2.12	2.07	2.01	1.97
10	2.98	2.54	2.35	2.24	2.16	2.08	2.03	1.96	1.93
15	2.85	2.40	2.20	2.09	2.01	1.92	1.87	1.80	1.77
20	2.77	2.33	2.12	2.01	1.93	1.84	1.78	1.71	1.68
25	2.73	2.28	2.07	1.96	1.88	1.78	1.73	1.65	1.62
30	2.70	2.25	2.04	1.92	1.84	1.74	1.69	1.61	1.57
40	2.66	2.20	1.99	1.87	1.79	1.69	1.63	1.55	1.52
50	2.64	2.18	1.97	1.84	1.76	1.66	1.60	1.52	1.48
75	2.60	2.14	1.93	1.80	1.72	1.61	1.55	1.47	1.42
100	2.59	2.12	1.91	1.78	1.70	1.59	1.52	1.44	1.39

Beispiel: Die Zufallsvariable $X \sim F_{40;5}$ und gesucht ist
$F(x_{0,95}) = 0,95 \Longrightarrow x_{0,95} = 4,46$

14.6 Verteilungsfunktion Φ der Standardnormalverteilung

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.500000	0.503989	0.507978	0.511966	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555670	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621720	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802337	0.805105	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823814	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.879000	0.881000	0.882977
1.2	0.884930	0.886861	0.888768	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903200	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935745	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959070	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965620	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999534	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999651
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999822	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967
4	0.999968	0.999970	0.999971	0.999972	0.999973	0.999974	0.999975	0.999976	0.999977	0.999978
4.1	0.999979	0.999980	0.999981	0.999982	0.999983	0.999983	0.999984	0.999985	0.999985	0.999986
4.2	0.999987	0.999987	0.999988	0.999988	0.999989	0.999989	0.999990	0.999990	0.999991	0.999991
4.3	0.999991	0.999992	0.999992	0.999993	0.999993	0.999993	0.999993	0.999994	0.999994	0.999994
4.4	0.999995	0.999995	0.999995	0.999995	0.999996	0.999996	0.999996	0.999996	0.999996	0.999996
4.5	0.999997	0.999997	0.999997	0.999997	0.999997	0.999997	0.999997	0.999998	0.999998	0.999998

Beispiele:

$$\Phi(0,27) = 0.606420$$
$$0,27 = 0,20 + 0,07$$

Wert aus Zeile mit 0,2 und Spalte mit 0,07

$$P(Z \le z) = \Phi(z)$$
$$P(Z \le 2,33) = \Phi(2,33) = 0,990097$$

$$\Phi(-z) = 1 - \Phi(z)$$
$$P(Z \le -0,6) = \Phi(-0,6) = 1 - \Phi(0,6) = 1 - 0,725747 = 0,274253$$

$$P(a \le Z \le b) = \Phi(b) - \Phi(a)$$
$$P(0,33 \le Z \le 2,33) = \Phi(2,33) - \Phi(0,33) = 0,360297$$

$$P(Z \ge z) = 1 - \Phi(z)$$
$$P(Z \ge 1,65) = 1 - \Phi(1,65) = 1 - 0,950529 = 0,049471$$

p-Quantil z_p von $N(0;1)$

$p = \Phi(z_p)$	$\Rightarrow$	z_p
0,001		−3,09
0,005		−2,58
0,010		−2,33
0,025		−1,96
0,050		−1,64
0,100		−1,28
0,900		+1,28
0,950		+1,64
0,975		+1,96
0,990		+2,33
0,995		+2,58
0,999		+3,09

Beispiel:

$$\Phi(z_{0,99}) = 0,99$$
$$\Longrightarrow z_{0,99} \approx +2,33$$